Example 4.31 $x^4+3x^3=5-x^5$

Is $x=1$ a solution? Is $x=2$ a solution?

Solution: Since $1^4+3\cdot 1^3=1+3=4$ and
$5-1^5=5-1=4$ then $x=1$ is a solution.
Since $2^4+3\cdot 2^3=16+3\cdot 8=16+24=40$ and
$5-2^5=5-31=-27$ then $x=2$ is not a solution.

Example 4.32 $y+\cos^2 x=x^2+1$.
Is $y=\sin^2 x+x^2$ a solution?
Is $y=x^2$ a solution?

Solution Since $(\sin^2 x+x^2)+\cos^2 x=x^2+\sin^2 x+\cos^2 x$
$=x^2+1$
and $x^2+1=x^2+1$ then $y=\sin^2 x+x^2$ is a solution.
Since $x^2+\cos^2 x\neq x^2+1$ then $y=x^2$ is not a solution.

Example 4.33 Verify that $y=e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2}=15y-2\frac{dy}{dx}$.

Solution $\frac{dy}{dx}=e^{3x}\cdot 3$ and $\frac{d^2y}{dx^2}=3e^{3x}\cdot 3=9e^{3x}$. Since

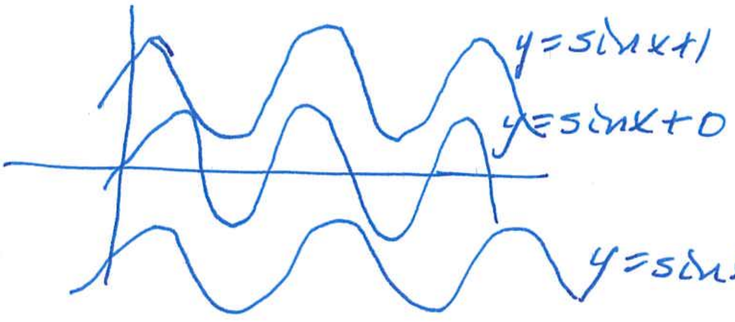
$15e^{3x}-2\cdot 3e^{3x}=(15-6)e^{3x}=9e^{3x}$ then
$y=e^{3x}$ is a solution to $\frac{d^2y}{dx^2}=15y-2\frac{dy}{dx}$.

Example 4.34 Solve $\frac{dy}{dx} = \cos x$ for y.

Solution Since $\frac{dy}{dx} = \cos x$ then

$\int \frac{dy}{dx}\,dx = \int \cos x\,dx$. So $\int dy = \int \cos x\,dx$.

So $y = \sin x + c$, where c is a constant.



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Example 4.35 $y = \sin x + 1$ is a particular solution of $\frac{dy}{dx} = \cos x$.

Example 4.36 Solve $\frac{dy}{dx} = \cos x$ with $y(\frac{\pi}{2}) = 3$.

Solution If $\frac{dy}{dx} = \cos x$ then $y = \sin x + c$.

If $3 = y(\frac{\pi}{2}) = \sin\frac{\pi}{2} + c = 1 + c$ then $c = 2$.

So $y = \sin x + 2$. //

Example 4.37 Solve $f''(x) = x$ with $f(1) = 2$ and $f'(0) = 1$.

Solution Since $\frac{d^2f}{dx^2} = x$ then $\int \frac{d^2f}{dx^2}dx = \int x\,dx$.

So $\frac{df}{dx} = \frac{1}{2}x^2 + c_1$, where c_1 is a constant.

Since $1 = f'(0) = \frac{1}{2}\cdot 0^2 + c_1$ then $c_1 = 1$.

So $\frac{df}{dx} = \frac{1}{2}x^2 + 1$. So $\int \frac{df}{dx}dx = \int (\frac{1}{2}x^2 + 1)\,dx$.

So $f = \frac{1}{2}\cdot\frac{1}{3}x^3 + x + c_2$, where c_2 is a constant.

Since $2 = f(1) = \frac{1}{6}1^3 + 1 + c_2 = 1 + \frac{1}{6} + c_2$ then

$$c_2 = 2 - 1 - \frac{1}{6} = 1 - \frac{1}{6} = \frac{5}{6}.$$

So $f = \frac{1}{6}x^3 + x + \frac{5}{6}$. //

Example 4.38 Verify that $f = x^2 + c\frac{1}{x}$, where c is a constant, is a solution of

$$f'(x) + \frac{1}{x}f(x) = 3x.$$

Solution If $f = x^2 + c\frac{1}{x}$ then $\frac{df}{dx} = 2x + c(-1)x^{-2}$

$$= 2x - \frac{c}{x^2}.$$

So $\frac{df}{dx} + \frac{1}{x} f = 2x - \frac{c}{x^2} + \frac{1}{x}\left(x^2 + c\frac{1}{x}\right)$

$$= 2x - \frac{c}{x^2} + x + \frac{c}{x^2} = 3x.$$

So f is a solution of $f' + \frac{1}{x} f = 3x$.