MAST10005 Slide
A. Ram 20.07.2019

Calculus 1

Goals / Skills

(1) To be able to do the questions on the exam and optimise your marks for your solutions.

(2) To learn about life and time management and life management.

(3) To dream and be a realist and achieve the dreams

(4) Work hard / Play hard
Be inspired and have fun.

MAST10005 Slide 1
A. Ram 20.07.2019

Number systems

Positive integers: $\mathbb{Z}_{>0} = \{1, 2, 3, 4, \ldots\}$

Nonnegative integers: $\mathbb{Z}_{\geq 0} = \{0, 1, 2, 3, 4, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational numbers:

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a \in \mathbb{Z},\ b \in \mathbb{Z}_{\neq 0} \text{ and } \frac{a}{b} = \frac{c}{d} \text{ if } ad = bc \right\}$$

Real numbers:

$$\mathbb{R} = \{ \pm a_\ell a_{\ell-1} \ldots a_1 a_0 . a_{-1} a_{-2} \ldots \mid \ell \in \mathbb{Z},\ a_i \in \{0, 1, \ldots 9\} \}$$

with a convention that if $a_k \neq 9$ then

$$\pm a_\ell \ldots a_{k+1} a_k 9999 \ldots = \pm a_\ell \ldots a_{k+1} (a_k + 1) 0000 \ldots$$

so that, for example, $0.9999\ldots = 1.0000\ldots$

MAST10005 Slide

A. Rom 20.07.2019

Intervals in $\mathbb{R}$

Let $a, b \in \mathbb{R}$ with $a < b$. Define

the set of … in … such that …

$$\mathbb{R}_{(a,b)} = \{x \in \mathbb{R} \mid a < x < b\} \qquad \mathbb{R}_{[a,b)} = \{x \in \mathbb{R} \mid a \leq x < b\}$$

$$\mathbb{R}_{(a,b]} = \{x \in \mathbb{R} \mid a < x \leq b\} \qquad \mathbb{R}_{[a,b]} = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

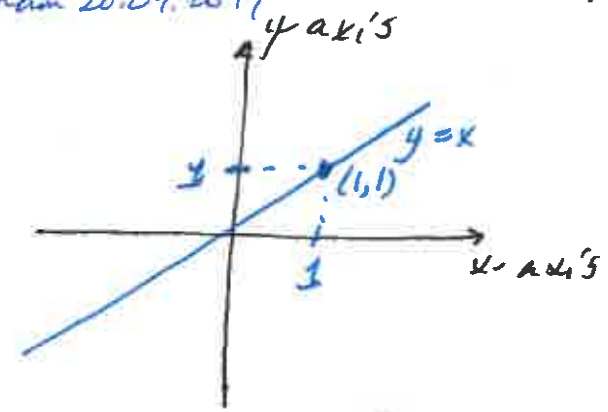
$$\mathbb{R}_{(a,\infty)} = \{x \in \mathbb{R} \mid a < x\} \qquad \mathbb{R}_{[a,\infty)} = \{x \in \mathbb{R} \mid a \leq x\}$$

$$\mathbb{R}_{(\infty,a)} = \{x \in \mathbb{R} \mid x < a\} \qquad \mathbb{R}_{(\infty,a]} = \{x \in \mathbb{R} \mid x \leq a\}.$$

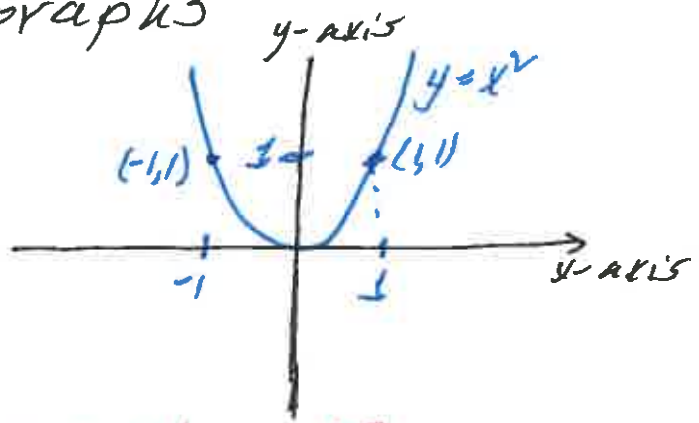
MAST10005 Slide

A. Ram 20.07.2019

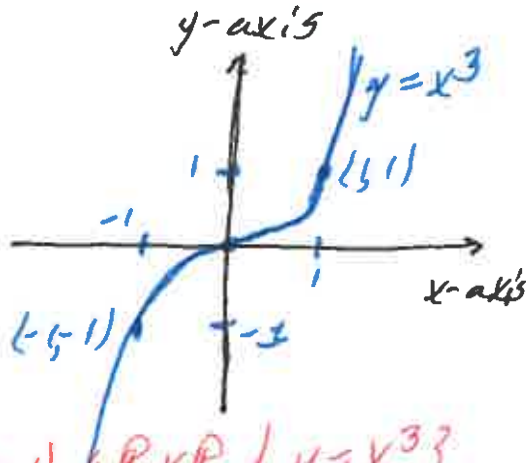
Some Graphs



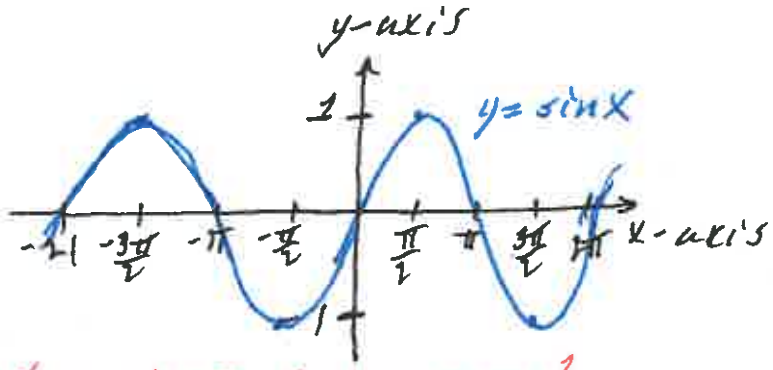
$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x\}$



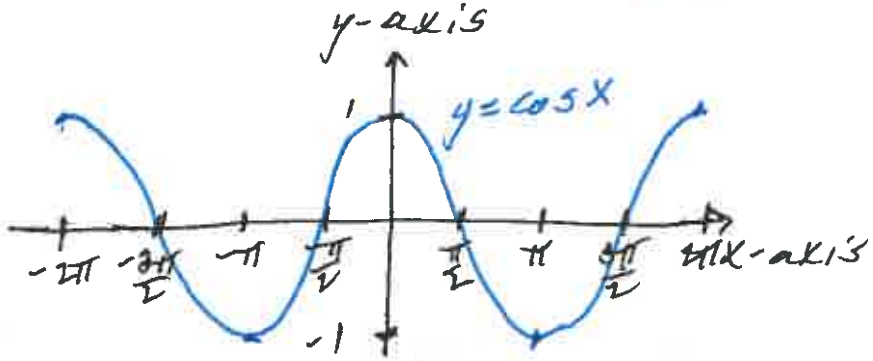
$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2\}$



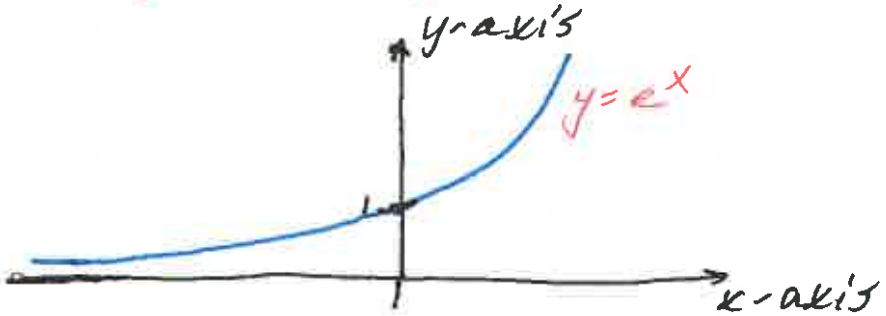
$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x^3\}$



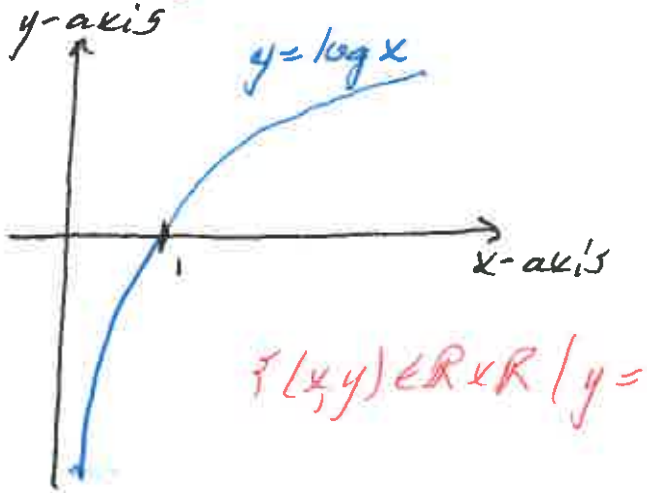
$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = \sin x\}$



$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = \cos x\}$



$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = e^x\}$



$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = \log x\}$

Lecture 1 30 July 2019
A. Kaur

Example 1.1 Translate from English to math or vice versa.

(a) The set of real numbers whose logarithm is positive.

(b) The set of integers whose square is even.

(c) $\{n \in \mathbb{N} / \sin(n) > 0\}$

Solution: (a) $\{x \in \mathbb{R} / \log(x) \in \mathbb{R}_{>0}\}$

(b) $\{x \in \mathbb{Z} / x^2 \text{ is even}\}$

$= \{x \in \mathbb{Z} / x^2 \in 2\mathbb{Z}\} = \{x \in \mathbb{Z} / x \in 2\mathbb{Z}\}$

$= 2\mathbb{Z}$

(c) The set of positive integers with positive sine.

Example 1.2 Give alternate descriptions of the following sets in math.

(a) The set of odd integers

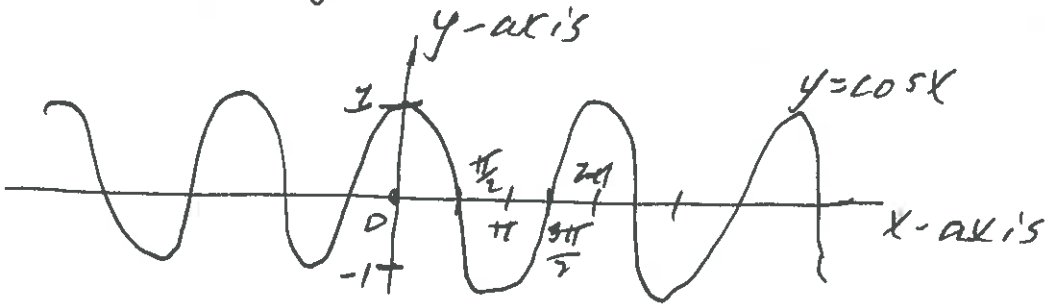
(b) $\{x \in \mathbb{R} \mid \cos x = 0\}$

(c) $\{x \in \mathbb{R} \mid \sin x = -1\}$.

Solution: (a) $\{x \in \mathbb{Z} \mid x \text{ is odd}\} = \{2k+1 \mid k \in \mathbb{Z}\}$

$= 2\mathbb{Z}+1$

(b) Since the graph of $y = \cos x$ is



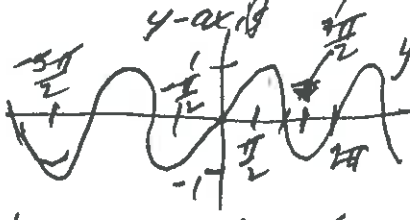
then

$\{x \in \mathbb{R} \mid \cos x = 0\} = \{\ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\}$

$= \{(2k+1)\frac{\pi}{2} \mid k \in \mathbb{Z}\} = \{k\pi + \frac{\pi}{2} \mid k \in \mathbb{Z}\}$

$= \frac{\pi}{2} + 2\mathbb{Z}\pi$.

(c) Since the graph of $y = \sin x$ is



then

$\{x \in \mathbb{R} \mid \sin x = -1\} = \{\ldots -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots\}$

$= \{\frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z}\} = -\frac{\pi}{2} + 2\pi\mathbb{Z}$.

Example 1.3 Define and graph $\mathbb{R}_{[a,b]}$ and $\mathbb{R}_{(a,b]}$

Solution

$\mathbb{R}_{[a,b)} = \{x \in \mathbb{R} \mid a \leq x < b\}$ has graph

$\mathbb{R}_{(a,b]} = \{x \in \mathbb{R} \mid a < x \leq b\}$ has graph

Example 1.4 Translate from English to math:
"The set of prime numbers less than 20"

Solution: $\{n \in \mathbb{Z}_{>0} \mid n < 20 \text{ and } n \text{ is prime}\}$

$= \{2, 3, 5, 7, 11, 13, 17, 19\}$

$= \{n \in \mathbb{Z}_{>0} \mid n \text{ divides } 9699690 \text{ and } n \text{ is prime}\}$

$= \{\text{prime factors of } 9699690\}$.

Example 1.5 Express $\{k\pi \mid k \in \mathbb{Z}\}$ in list of elements form and graph it.

Solution: $\{k\pi \mid k \in \mathbb{Z}\} = \{\ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$