# Assignment 2

MAST90097 Algebraic Geometry
Semester II 2018
Lecturer: Arun Ram
to be turned in before 2pm on 7 September 2018

**1.** Carefully, precisely and accurately (i.e. in proof machine) define projective space.

**2.** Looking at $\mathbb{P}^1$.

(a) Explain precisely (with proof) how $\mathbb{P}^1$ is two copies of $\mathbb{C}$ glued together.

(b) Explain precisely (with proof) how $\mathbb{P}^1$ is isomorphic to $S^2$.

(c) Explain precisely (with proof) how $\mathbb{P}^1$ is a one point compactification of $\mathbb{C}$.

(d) Explain precisely (with proof) how $\mathbb{P}^1$ is $GL_2(\mathbb{C})/B$.

(e) Explain precisely (with proof) how $\mathbb{P}^1$ is a quotient of $\mathbb{C}^2 - \{0\}$.

(f) Determine (with proof) the coordinate ring of $\mathbb{P}^1$.

(g) Determine (with proof) the structure sheaf of $\mathbb{P}^1$.

(h) Show that $\mathbb{P}^1$ is compact.

(i) Show that $\mathbb{P}^1$ is a Riemann surface of genus 0.

(j) Construct the line bundles on $\mathbb{P}^1$.

(k) Complete the classification of line bundles on $\mathbb{P}^1$ by showing that the line bundles you constructed in (j) form a set of representatives of the isomorphism classes of line bundles on $\mathbb{P}^1$.

(l) For each line bundle $\mathcal{L}$ on $\mathbb{P}^1$ determine the holomorphic sections of $\mathcal{L}$.