Tutorial: Week 12 Preparing for the exam

Proofs to do in proofmachine (without cheating and looking at the notes).

Task 1

Theorem Let (X, d) be a metric space. Let $\mathcal{T}_X$ be the metric space topology on X. Let $A \subseteq X$. Then

$$\overline{A} \subseteq \left\{ z \in X \;\middle|\; \begin{array}{l} \text{there exists a sequence } (a_1, a_2, \ldots) \text{ in } A \\ \text{such that } z = \lim_{n \to \infty} a_n \end{array} \right\},$$

where $\overline{A}$ is the closure of A in X.

Task 2

Theorem Let $(V, \| \|)$ and $(W, \| \|)$ be normed vector spaces. Carefully define $B(V, W)$ and show that if W is complete then $B(V, W)$ is complete.

Task 3

Theorem Let (X, d) be a metric space. Let $A \subseteq X$.

Prove

$$\begin{array}{ccccc}
A \text{ is cover compact} & \Rightarrow & A \text{ is ball compact} & \Rightarrow & A \text{ is bounded} \\
\Downarrow\Uparrow & & \Leftarrow \quad + & & \\
A \text{ is sequentially compact} & \Rightarrow & A \text{ is Cauchy compact} & \Rightarrow & A \text{ closed in } X
\end{array}$$

Task 4

<u>Proposition</u> Let $(X, \mathcal{T})$ be a topological space. Let $E \subseteq X$.

(a) The interior of E is the set of interior points of E.

(b) The closure of E is the set of close points of E.

Be sure to very carefully define the terms interior, closure, interior points and close points before embarking on the proof.

Task 5

<u>Theorem</u> Let $f: S \to T$ be a function. The inverse function to f exists if and only if f is bijective.

Be sure to very carefully define the terms function, inverse function to f and bijective before embarking on the proof.