(6b) To show: If $A \subseteq \mathbb{R}_{\geq 0}$ then
A is compact if and only if A is closed and bounded.

Assume $A \subseteq \mathbb{R}_{\geq 0}$

To show: (6a) If A is compact then
A is closed and bounded.
(6b) If A is closed and bounded then A is compact.

For (6a) let us simply quote Theorem 6.0.1 in the notes, which is proved in §10.17, specifically the proof of Theorem 10.17.1 parts (a), (b) and (c) and (d) and (e).

If there is more time available then we will work through the proofs given in the notes to check them.

(6b) To show: If A is closed and bounded then
A is compact.
Assume A is closed and bounded.
To show: A is compact.
Since A is closed in $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{\geq 0}$ is complete then, by §6.3.1 exercise (14), which is proved in (cb) of Theorem 10.8.1, then A is complete.

Since A is bounded then, by the proof of (ca) of Theorem 10.8.1, then
A is ball compact.

Finally, by the proof of Theorem 10.18.1,
A is sequentially compact

The flow chart here is: A closed and bounded $\Rightarrow$ A sequentially compact

A closed in $\mathbb{R}_{\geq 0}$ + $\mathbb{R}_{\geq 0}$ complete $\xRightarrow[\text{Theorem 10.8.1 part (bb)}]{}$ A complete

A bounded in $\mathbb{R}_{\geq 0}$ $\xRightarrow[\text{Theorem 10.8.1 part (ca)}]{}$ A is ball compact.

A complete + A ball compact $\xRightarrow[\text{Theorem 10.14.1 } (\Leftarrow)]{}$ A sequentially compact

The flow chart for A sequentially compact $\Rightarrow$ A closed and bounded is

A sequentially compact $\xRightarrow[\text{Theorem 10.17.1 (b)}]{}$ A Cauchy compact

A Cauchy compact $\xRightarrow[\text{Theorem 10.17.1 (e)}]{}$ A is closed

A sequentially compact $\xRightarrow[\text{Theorem 10.17.1 (ab)}]{}$ A cover compact

A cover compact $\Rightarrow$ A ball compact
Theorem 10.17.1 (c).

A ball compact $\Rightarrow$ A is bounded
Theorem 10.17.1 (d).