Functional analysis: Lecture 1 28 July 2014

Information

(1) Google: Arun Ram

(2) Contact and availability: thoughts and advice page.

(3) SSLC Representatives.

(4) Scribe for

HWs, Vocabulary and Examples.

(5) Learning mathematics

(6) Books: (a) Bressan

(b) Others?

(7) Schedule: Times away – Thursday 11-1 in G03, Talks on Fri.

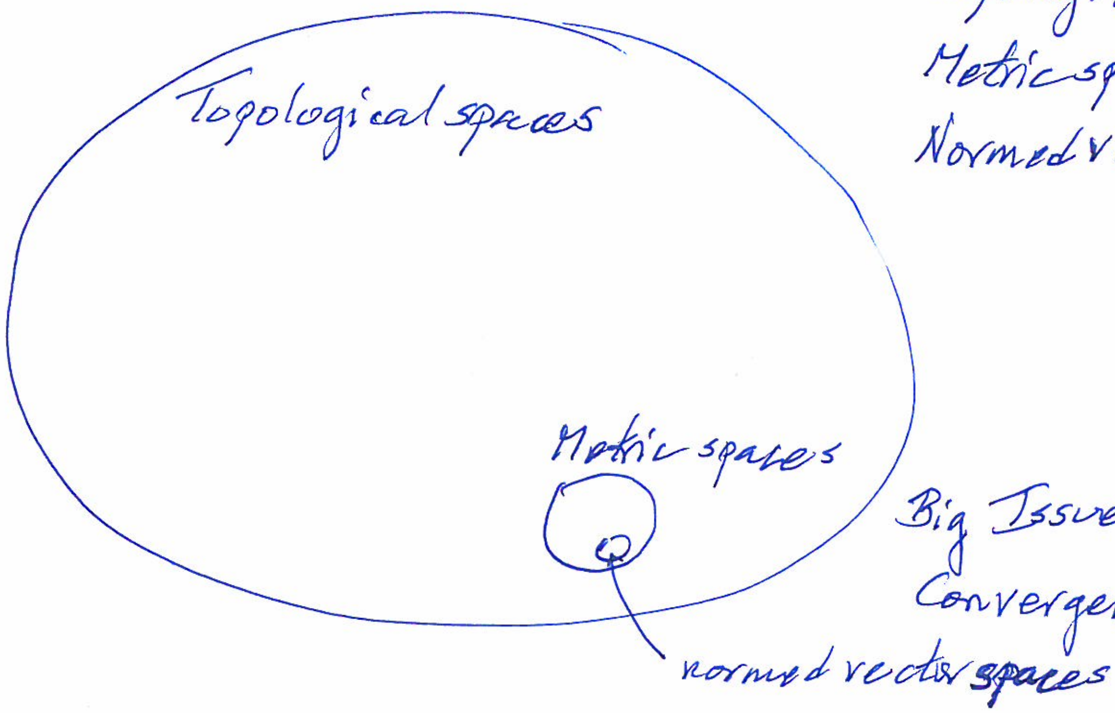
(8) Homework and Exams

Structures

Topological spaces

Metric spaces

Normed vector spaces



Big Issue:

Convergence

Examples

$\mathbb{R}_{\geq 0}$, $\mathbb{Q}_{\geq 0}$, $\mathbb{Z}_{\geq 0} = \mathbb{Z}_{>0}$

$\mathbb{R}^n = L^1([1,n]_{\mathbb{Z}})$, $\mathbb{R}^n = L^p([1,n]_{\mathbb{Z}})$, $\mathbb{R}^n = L^\infty([1,n]_{\mathbb{Z}})$

$\mathbb{R}^\infty = L^1(\mathbb{Z}_{>0}) = \ell^1$, $\mathbb{R}^\infty = L^p(\mathbb{Z}_{>0}) = \ell^p$, $\mathbb{R}^\infty = L^\infty(\mathbb{Z}_{>0}) = \ell^\infty$

<u>HW</u>: Are these right?

$L^1([0,1])$, $L^p([0,1])$ and $L^\infty([0,1])$

$L^1(\mathcal{U})$, $L^p(\mathcal{U})$ and $L^\infty(\mathcal{U})$.

<u>Big point</u>: These are all spaces of functions.

<u>HW</u>: How do we define completeness on topological spaces?

<u>HW</u>: Why are "for all" and "for each" not used by proof machine?

Bressan 23.02.2014

From Bressan

<u>Definition</u>: Let X and Y be metric spaces. Let $\alpha \in (0,1]_{\mathbb{R}}$.

- A <u>Lipschitz continuous function from X to Y</u> is a function $f: X \to Y$ such that there exists $C \in \mathbb{R}_{\geq 0}$ such that

  if $x_1, x_2 \in X$ then $d(f(x_1), f(x_2)) \leq C \cdot d(x_1, x_2)$.

- A <u>Hölder continuous function of exponent α from X to Y</u> is a function $f: X \to Y$ such that there exists $C \in \mathbb{R}_{\geq 0}$ such that

  if $x_1, x_2 \in X$ then $d(f(x_1), f(x_2)) \leq C \cdot d(x_1, x_2)^\alpha$.

<u>Definition</u>: Let X be a metric space.

- The metric space X is <u>precompact</u> if X satisfies: if $\varepsilon \in \mathbb{R}_{>0}$ then there exists $\ell \in \mathbb{Z}_{>0}$ and $x_1, x_2, \ldots, x_\ell \in X$ such that

  $$X = B_\varepsilon(x_1) \cup \cdots \cup B_\varepsilon(x_\ell).$$

<u>Theorem</u> Let X be a metric space. The following are equivalent:

(a) X is compact

(b) X is precompact and complete

(c) If $x_1, x_2, \ldots$ is a sequence in X then there exists a subsequence $x_{n_1}, x_{n_2}, \ldots$ which converges.