



**Math 541**  
**Modern Algebra**  
**A first course in Abstract Algebra** Fall 2007  
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**Homework 9: Due November 8, 2007**

**To grade: 4, 7, 10.**

1. Let  $R$  be a commutative ring. Define prime ideal and maximal ideal and give some examples.
2. Show that every field is an integral domain and give an example of an integral domain that is not a field.
3. Let  $R$  be a commutative ring. Show that every maximal ideal is prime.
4. Give an example of a prime ideal that is not maximal.
5. Let  $R$  be a ring (not necessarily commutative). Define maximal ideal.
6. Define simple ring.
7. Let  $R$  be a ring (not necessarily commutative) and let  $I$  be an ideal of  $R$ . Show that  $I$  is a maximal ideal of  $R$  if and only if  $R/I$  is a simple ring.
8. Show that every division ring is a simple ring.
9. Give an example of a simple ring that is not a division ring.
10. Show that if  $R$  is a commutative simple ring then  $R$  is a field.
11. Let  $R$  be a commutative ring. Show that the two different definitions of maximal ideal are equivalent.