University of Wisconsin-Madison
Mathematics Department

Math 541
Modern Algebra
A first course in Abstract Algebra
Lecturer: Arun Ram

Fall 2007

Homework 7: Due October 24, 2007

To grade: 4, 11, 15.

1. Define homomorphism, kernel and image and give some examples.
2. Let $f : G \to H$ be a group homomorphism. Show that $\ker f$ is a normal subgroup of G.
3. Let $f : G \to H$ be a group homomorphism. Show that $\frac{G}{\ker f} \simeq \mathrm{im}\, f$.
4. Let $f : G \longrightarrow H$ be a group homomorphism. Show that f is injective if and only if $\ker f = \{1\}$.
5. Let $f : G \longrightarrow H$ be a group homomorphism. Show that f is surjective if and only if $\mathrm{im}\, f = H$.
6. Define ring, subgroup, coset, R/I and ideal and give some examples.
7. Let I be a subgroup of a ring R. Show that the operation on R/I given by
$$(r_1 + I)(r_2 + I) = r_1 + r_2 + I$$
is well defined.
8. Let I be a subgroup of a ring R. Show that the operation on R/I given by
$$(r_1 + I)(r_2 + I) = r_1 r_2 + I$$
is well defined then I is an ideal of R.
9. Let I be a subgroup of a ring R. Show that if I is an ideal of R then the operation on R/I given by
$$(r_1 + I)(r_2 + I) = r_1 r_2 + I$$
is well defined.
10. Let I be an ideal of a ring R. Show that R/I with operations given by

$$(r_1 + I) + (r_2 + I) = (r_1 + r_2) + I \qquad \text{and} \qquad (r_1 + I)(r_2 + I) = r_1 r_2 + I$$

is a ring.

11. Determine the ideals of $\mathbb{Z}$.
12. Let $f : R \longrightarrow A$ be a ring homomorphism. Show that $\ker f$ is an ideal of R.
13. Let $f : R \longrightarrow A$ be a ring homomorphism. Show that $\operatorname{im} f$ is a subgroup of R.
14. Let $f : R \longrightarrow A$ be a ring homomorphism. Show that $\operatorname{im} f$ is an ideal of A.
15. Let $f : R \to A$ be a ring homomorphism. Show that $\frac{R}{\ker f} \simeq \operatorname{im} f$.
16. Let $f : R \longrightarrow A$ be a ring homomorphism. Show that f is injective if and only if $\ker f = \{0\}$.
17. Let $f : R \longrightarrow A$ be a ring homomorphism. Show that f is surjective if and only if $\operatorname{im} f = A$.