



**Math 541**  
**Modern Algebra**  
**A first course in Abstract Algebra** Fall 2007  
**Lecturer: [Arun Ram](#)**

[University of Wisconsin-Madison](#)  
[Mathematics Department](#)

---

**Homework 5: Due October 10, 2007**

1. Let  $d \in \mathbb{Z}_{\geq 0}$ . Show that the set  $d\mathbb{Z}$  of multiples of  $d$  is a subgroup of  $\mathbb{Z}$ .
2. Show that if  $H$  is a subgroup of  $\mathbb{Z}$  then there exists a positive integer  $d$  such that  $H = d\mathbb{Z}$ .
3. Let  $d_1$  and  $d_2$  be positive integers. Show that  $d_2\mathbb{Z} \subseteq d_1\mathbb{Z}$  if and only if  $d_1$  divides  $d_2$ .
4. Make a list of all the subgroups of  $\mathbb{Z} / 110\mathbb{Z}$ .
5. Make a list of all the subgroups of the Klein 4 group.
6. Make a list of all the subgroups of the symmetric group  $S_3$ .