HW1 Problem 6

Prove: If A and B are $n \times m$ matrices then $A+B=B+A$.

Assume A and B are $n \times m$ matrices.

To show: $A+B=B+A$.

To show: $(A+B)_{ij} = (B+A)_{ij}$.

$$(A+B)_{ij} = A_{ij} + B_{ij} = B_{ij} + A_{ij} = (B+A)_{ij}.$$

So $A+B=B+A$.

HW1 Problem 7

Prove: If A, B and C are $n \times m$ matrices then $A+(B+C) = (A+B)+C$.

Assume A, B and C are $n \times m$ matrices.

To show: $(A+(B+C))_{ij} = ((A+B)+C)_{ij}$

$$\begin{aligned}(A+(B+C))_{ij} &= A_{ij} + (B+C)_{ij} \\ &= A_{ij} + (B_{ij} + C_{ij}) \\ &= (A_{ij} + B_{ij}) + C_{ij} \\ &= (A+B)_{ij} + C_{ij} = ((A+B)+C)_{ij}.\end{aligned}$$

So $A+(B+C) = (A+B)+C$.

<u>HW1 Problem 8</u> If A is an $m\times n$ matrix, B is an $n\times p$ matrix and C is a $p\times r$ matrix then $(AB)C = A(BC)$.

<u>Proof</u>

Assume A is an $m\times n$ matrix, B is an $n\times p$ matrix and C is a $p\times r$ matrix.

To show: $(AB)C = A(BC)$.

To show: $((AB)C)_{ij} = (A(BC))_{ij}$.

$$
\begin{aligned}
((AB)C)_{ij} &= (AB)_{i1}C_{1j} + (AB)_{i2}C_{2j} + \cdots + (AB)_{ip}C_{pj} \\
&= (A_{i1}B_{11} + A_{i2}B_{21} + \cdots + A_{in}B_{nj})C_{1j} \\
&\quad + (A_{i1}B_{12} + A_{i2}B_{22} + \cdots + A_{in}B_{n2})C_{2j} \\
&\quad + \cdots \\
&\quad + (A_{i1}B_{1p} + A_{i2}B_{2p} + \cdots + A_{in}B_{np})C_{pj}. \\
&= A_{i1}B_{11}C_{1j} + A_{i2}B_{21}C_{1j} + \cdots + A_{in}B_{n1}C_{1j} \\
&\quad + A_{i1}B_{12}C_{2j} + A_{i2}B_{22}C_{2j} + \cdots + A_{in}B_{n2}C_{2j} \\
&\quad + \cdots \\
&\quad + A_{i1}B_{1p}C_{pj} + A_{i2}B_{2p}C_{pj} + \cdots + A_{in}B_{np}C_{pj}
\end{aligned}
$$

This is the left hand side (LHS).

$$RHS = (A(BC))_{ij} = A_{i1}(BC)_{1j} + A_{i2}(BC)_{2j} + \cdots + A_{in}(BC)_{nj}$$

$$\begin{aligned}
&= A_{i1}(B_{11}C_{1j} + B_{12}C_{2j} + \cdots + B_{1p}C_{pj}) \\
&\quad + A_{i2}(B_{21}C_{1j} + B_{22}C_{2j} + \cdots + B_{2p}C_{pj}) \\
&\quad + \cdots \\
&\quad + A_{in}(B_{n1}C_{1j} + B_{n2}C_{2j} + \cdots + B_{np}C_{pj})
\end{aligned}$$

$$\begin{aligned}
&= A_{i1}B_{11}C_{1j} + A_{i1}B_{12}C_{2j} + \cdots + A_{i1}B_{1p}C_{pj} \\
&\quad + A_{i2}B_{21}C_{1j} + A_{i2}B_{22}C_{2j} + \cdots + A_{i2}B_{2p}C_{pj} \\
&\quad + \cdots \\
&\quad + A_{in}B_{n1}C_{1j} + A_{in}B_{n2}C_{2j} + \cdots + A_{in}B_{np}C_{pj}
\end{aligned}$$

which is equal to the left hand side.

So $((AB)C)_{ij} = (A(BC))_{ij}$.

So $(AB)C = A(BC)$.

<u>HW1 Problem 9</u> Prove: If A and B are $m \times n$ matrices and C is an $n \times p$ matrix then $(A+B)C = AC+BC$.

Assume A and B are $m \times n$ matrices and C is an $n \times p$ matrix.

To show: $((A+B)C)_{ij} = (AC+BC)_{ij}$.

$$((A+B)C)_{ij} = (A+B)_{i1}C_{1j} + (A+B)_{i2}C_{2j} + \cdots + (A+B)_{in}C_{nj}.$$

$$= (A_{i1}+B_{i1})C_{1j} + (A_{i2}+B_{i2})C_{2j} + \cdots + (A_{in}+B_{in})C_{nj}$$

$$= A_{i1}C_{1j} + B_{i1}C_{1j} + A_{i2}C_{2j} + B_{i2}C_{2j} + \cdots + A_{in}C_{nj} + B_{in}C_{nj}$$

$$= A_{i1}C_{1j} + A_{i2}C_{2j} + \cdots + A_{in}C_{nj}$$
$$+ B_{i1}C_{1j} + B_{i2}C_{2j} + \cdots + B_{in}C_{nj}$$

$$= (AC)_{ij} + (BC)_{ij} = (AC+BC)_{ij}.$$

So $(A+B)C = AC+BC$.

<u>HW1 Problem 11</u> Prove: If r and s are numbers and A is a matrix then $r(sA) = (rs)A$.

Assume r and s are numbers and A is a matrix.

To show: $r(sA) = (rs)A$

To show: $(r(sA))_{ij} = ((rs)A)_{ij}$.

$$(r(sA))_{ij} = r(sA)_{ij} = rsA_{ij} = ((rs)A)_{ij}.$$

So $r(sA) = (rs)A$.

<u>HW1 Problem 12</u> Prove: If r and s are numbers and A is a matrix then $(r+s)A = rA + sA$.

Assume r and s are numbers and A is a matrix.

To show: $(r+s)A = rA + sA$.

To show: $((r+s)A)_{ij} = (rA+sA)_{ij}$.

~~To show: r?~~

$$((r+s)A)_{ij} = (r+s)A_{ij} = rA_{ij} + sA_{ij}$$
$$= (rA)_{ij} + (sA)_{ij} = (rA+sA)_{ij}.$$

So $(r+s)A = rA + sA$.

<u>HW1 Problem 13</u> Prove: If r is a number and A and B are matrices then $r(A+B) = rA + rB$.

Assume r is a number and A and B are matrices.

To show: $r(A+B) = rA + rB$.

To show: $(r(A+B))_{ij} = (rA+rB)_{ij}$

$$(r(A+B))_{ij} = r(A+B)_{ij} = rA_{ij} + rB_{ij}$$
$$= (rA)_{ij} + (rB)_{ij} = (rA+rB)_{ij}.$$

So $r(A+B) = rA + rB$.

HW1 Problem 14 Prove: If r is a number and A and B are matrices then $A(rB) = r(AB) = (rA)B$.

Assume r is a number and A and B are matrices.

To show: (a) $A(rB) = r(AB)$

(b) $r(AB) = (rA)B$.

(a) To show: $(A(rB))_{ij} = (r(AB))_{ij}$.

$$(A(rB))_{ij} = A_{i1}(rB)_{1j} + A_{i2}(rB)_{2j} + \cdots + A_{in}(rB)_{nj}$$

$$= A_{i1} r B_{1j} + A_{i2} r B_{2j} + \cdots + A_{in} r B_{nj}$$

$$= r(A_{i1}B_{1j} + A_{i2}B_{2j} + \cdots + A_{in}B_{nj})$$

$$= r(AB)_{ij} = (r(AB))_{ij}.$$

(b) To show: $(r(AB))_{ij} = ((rA)B)_{ij}$.

$$(r(AB))_{ij} = r(AB)_{ij} = r(A_{i1}B_{1j} + \cdots + A_{in}B_{nj})$$

$$= rA_{i1}B_{1j} + rA_{i2}B_{2j} + \cdots + rA_{in}B_{nj}.$$

$$= (rA)_{i1}B_{1j} + (rA)_{i2}B_{2j} + \cdots + (rA)_{in}B_{nj}.$$

$$= ((rA)B)_{ij}.$$

So $A(rB) = r(AB) = (rA)B$.

HW1 Problem 15 Prove: If A is a matrix then $(A^t)^t = A$.

Assume A is a matrix.

To show: $(A^t)^t = A$.

To show: $((A^t)^t)_{ij} = A_{ij}$.

$$((A^t)^t)_{ij} = (A^t)_{ji} = A_{ij}.$$

So $(A^t)^t = A$.

HW1 Problem 16 Prove: If A and B are $m \times n$ matrices then $(A+B)^t = A^t + B^t$.

Assume A and B are $m \times n$ matrices.

To show: $(A+B)^t = A^t + B^t$.

To show: $((A+B)^t)_{ij} = (A^t + B^t)_{ij}$.

$$LHS = ((A+B)^t)_{ij} = (A+B)_{ji} = A_{ji} + B_{ji}.$$

$$RHS = (A^t + B^t)_{ij} = (A^t)_{ij} + (B^t)_{ij} = A_{ji} + B_{ji}.$$

So $(A+B)^t = A^t + B^t$.

HW 1 Problem 17 Prove: If A is an $m \times n$ matrix and B is an $n \times p$ matrix then $(AB)^t = B^t A^t$.

Assume A is an $m \times n$ matrix and B is an $n \times p$ matrix.

To show: $(AB)^t = B^t A^t$

To show: $((AB)^t)_{ij} = (B^t A^t)_{ij}$

$$LHS = ((AB)^t)_{ij} = (AB)_{ji}$$
$$= A_{j1} B_{1i} + A_{j2} B_{2i} + \cdots + A_{jn} B_{ni}$$

$$RHS = (B^t A^t)_{ij}$$
$$= (B^t)_{i1} (A^t)_{1j} + (B^t)_{i2} (A^t)_{2j} + \cdots + (B^t)_{in} (A^t)_{nj}$$
$$= B_{1i} A_{j1} + B_{2i} A_{j2} + \cdots + B_{ni} A_{jn}$$
$$= A_{j1} B_{1i} + A_{j2} B_{2i} + \cdots + A_{jn} B_{ni}.$$

So $LHS = RHS$.

So $(AB)^t = B^t A^t$.

HW1 Problem 18 Prove: If r is a number and A is a matrix then $(rA)^t = rA^t$.

Assume r is a number and A is a matrix.

To show: $(rA)^t = rA^t$.

To show: $((rA)^t)_{ij} = (rA^t)_{ij}$.

$$((rA)^t)_{ij} = (rA)_{ji} = r A_{ji} = r (A^t)_{ij} = (rA^t)_{ij}.$$

So $(rA)^t = rA^t$.

HW1 Problem 19 Give an example of two 5×5 matrices A and B such that AB is not equal to BA.

Let $a = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$

Then

$$ab = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix} \text{ and}$$

$$ba = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} \text{ so that } ab \neq ba.$$

Let

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Then

$$AB = \begin{pmatrix} 7 & 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ and

$$BA = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 7 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ so that $AB \neq BA$.

<u>HW1 Problem 20</u> Give an example of two 5×5 matrices A and B such that no entries of A or B are 0, A is not equal to B and $AB = BA$.

Let $$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$ and $$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 4 \\ 3 & 4 & 5 & 4 & 2 \\ 4 & 5 & 4 & 2 & 3 \\ 5 & 4 & 2 & 3 & 4 \end{pmatrix}$$

Then

$$AB = \begin{pmatrix} 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 & 15 \end{pmatrix} = BA.$$

HW3 Problem 6 Show that $tr(A+B) = tr(A) + tr(B)$

$$tr(A+B) = (A+B)_{11} + (A+B)_{22} + \cdots + (A+B)_{nn}$$

$$= A_{11} + B_{11} + A_{22} + B_{22} + \cdots + A_{nn} + B_{nn}$$

$$= A_{11} + A_{22} + \cdots + A_{nn} + B_{11} + B_{22} + \cdots + B_{nn}$$

$$= tr\, A + tr\, B.$$

HW3 Problem 8 Suppose that $\det(A) = 5$. Show that $\det(A^{-1}) = \frac{1}{5}$.

Assume $\det(A) = 5$

To show: (a) A^{-1} exists

(b) $\det A = 5$.

(a) By row reduction

$$A = R_1 R_2 \cdots R_\ell E$$

where $R_1, \dots, R_\ell$ are row operations and E is in echelon form.

Since the $R_1, \dots, R_\ell$ are invertible,

A is invertible if E is invertible

So A is invertible if E is the identity and

~~If~~ A is not invertible if E has a zero row.

If E has a zero row then

$$\det(A) = \det(R_1) \cdots \det(R_\ell) \det(E) = \det(R_1) \cdots \det(R_\ell) \cdot 0 = 0$$

So, since $\det A = 5$, A is invertible.

(b) $\det(AA^{-1}) = \det(A)\det(A^{-1})$

and $\det(AA^{-1}) = \det(I) = 1$.

So $\det(A^{-1}) = \dfrac{1}{\det A} = \dfrac{1}{5}$.

HW3 Problem 9 Define $n!$ and show that the number of permutations in S_n is $n!$

Let n be a positive integer.

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1.$$

A permutation matrix is a matrix with exactly one 1 in each row and each column and all other entries 0.

S_n is the set of $n\times n$ permutation matrices.

To construct an $n\times n$ permutation matrix

(a) there are n choices of where to put the 1 in the first row

(b) once the first row is chosen, there are $n-1$ choices of where to put the 1 in the second row,

(c) once the first two rows are chosen there are $n-2$ choices of where to put the 1 in the third row,

$\vdots$

Finally, once the first $n-1$ rows are chosen there is only 1 possibility for where to put the 1 in the last row.

Hence, the total number of possibilities is

$$n(n-1)(n-2)\cdots 2\cdot 1 = n!$$

So S_n contains $n!$ matrices.

HW3 Problem 10 Let A be an $n\times n$ matrix. Show that if A is invertible then $\det(A)\neq 0$.

Proof:
Assume A is an $n\times n$ matrix and A is invertible.
To show: $\det(A)\neq 0$.
Since A is invertible

$$\det(A)\det(A^{-1}) = \det(AA^{-1}) = \det(I) = 1$$

This is impossible if $\det(A)=0$, so it must be that $\det(A)\neq 0$.

HW3 Problem 11 Let A be an $n\times n$ matrix. Show that if $\det A\neq 0$ then A is invertible.

This was done in part (a) of the proof in HW3 Problem 8.

<u>HW3 Problem 13</u> Show that $tr(AB) = tr(BA)$

$$tr(AB) = (AB)_{11} + (AB)_{22} + \cdots + (AB)_{nn}$$

$$= A_{11}B_{11} + A_{12}B_{21} + \cdots + A_{1n}B_{n1}$$
$$+ A_{21}B_{12} + A_{22}B_{22} + \cdots + A_{2n}B_{n2}$$
$$+ \cdots$$
$$+ A_{n1}B_{1n} + A_{n2}B_{2n} + \cdots + A_{nn}B_{nn}$$

and

$$tr(BA) = (BA)_{11} + (BA)_{22} + \cdots + (BA)_{nn}$$

$$= B_{11}A_{11} + B_{12}A_{21} + \cdots + B_{1n}A_{n1}$$
$$+ B_{21}A_{12} + B_{22}A_{22} + \cdots + B_{2n}A_{n2}$$
$$+ \cdots$$
$$+ B_{n1}A_{1n} + B_{n2}A_{2n} + \cdots + B_{nn}A_{nn}$$

$$= A_{11}B_{11} + A_{21}B_{12} + \cdots + A_{n1}B_{1n}$$
$$+ A_{12}B_{21} + A_{22}B_{22} + \cdots + A_{n2}B_{2n}$$
$$+ \cdots$$
$$+ A_{1n}B_{n1} + A_{2n}B_{n2} + \cdots + A_{nn}B_{nn}$$

so $LHS = RHS$.

So $tr(AB) = tr(BA)$

HW2 Problem 2 Show that there is a unique $m \times n$ matrix A such that $A+B=B$ for all $m \times n$ matrices B.

Proof To show: (a) A exists
(b) A is unique.

(a) Let A be the matrix with all entries 0.

To show: If B is a matrix then $A+B=B$.

Assume B is an $m \times n$ matrix.

To show: $A+B=B$.

To show: $(A+B)_{ij}=B_{ij}$.

$$(A+B)_{ij} = A_{ij}+B_{ij} = 0+B_{ij} = B_{ij},$$

since all entries of A are 0.

(b) To show: If A is some matrix such that $A+B=B$ for all B then A has all entries 0.

Assume A is some matrix such that $A+B=B$ for all B.

Then, we know $A+0=0$.

So $(A+0)_{ij}=0_{ij}$.

So $A_{ij}+0_{ij}=0_{ij}$.

So $A_{ij}+0=0$.

So $A_{ij}=0$.

So all entries of A are 0.

HW2 Problem 3 Show that there is a unique $n\times n$ matrix such that $AB = B$ for all $n\times n$ matrices B.

Proof To show: (a) A exists
(b) A is unique.

(a) Let A be the matrix with $A_{ii}=1$ and $A_{ij}=0$ if $i\neq j$.

To show: If B is an $n\times n$ matrix then $AB=B$.

Assume B is an $n\times n$ matrix.

To show: $AB=B$.

To show: $(AB)_{ij}=B_{ij}$.

$$(AB)_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j} + \cdots + A_{in}B_{nj}.$$

$$= 0\cdot B_{1j} + 0 B_{2j} + \cdots + 1\cdot B_{ij} + \cdots + 0 B_{nj}$$

since $A_{ik}=0$ unless $k=i$ and $A_{ii}=1$.

So $(AB)_{ij} = 1\cdot B_{ij} = B_{ij}$.

So $AB=B$.

(b) To show: If A is a matrix such that $A\cdot B = B$ for all B then $A_{ii}=1$ and $A_{ij}=0$ for $i\neq j$.

Assume A is a matrix such that $AB=B$ for all B.

Then $A \cdot 1 = 1$.

So $(A\cdot 1)_{ij} = 1_{ij}$.

So $A_{i1} 1_{1j} + A_{i2} 1_{2j} + \cdots + A_{in} 1_{nj} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j. \end{cases}$

So $A_{i1}\cdot 0 + A_{i2}\cdot 0 + \cdots + A_{ij}\cdot 1 + \cdots + A_{in}\cdot 0 = \begin{cases} 0, & \text{if } i\neq j \\ 1, & \text{if } i=j. \end{cases}$

So $A_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$. //.

<u>HW2 Problem 4</u> Let A be a matrix. Show that there is a unique matrix B such that $B+A=0$

<u>Proof</u> Assume A is a matrix.

To show: (a) There exists B such that $B+A=0$.

(b) There is a unique B such that $B+A=0$.

(a) Let B be the matrix with entries

$$B_{ij} = -A_{ij}.$$

To show: $B+A=0$

To show: $(B+A)_{ij} = 0_{ij}$.

$(B+A)_{ij} = B_{ij} + A_{ij} = -A_{ij} + A_{ij} = 0 = 0_{ij}$.

So $B+A=0$.

(b) To show: If B is a matrix such that $B+A=0$ then B has entries $B_{ij}=-A_{ij}$.

Assume B is a matrix such that $B+A=0$.

To show: $B_{ij}=-A_{ij}$.

We know $B+A=0$.

So $(B+A)_{ij}=0_{ij}$.

So $B_{ij}+A_{ij}=0$.

So $B_{ij}=-A_{ij}$ //.

<u>HW2 Problem 5</u> Give an example of a nonzero 5×5 matrix A such that there does not exist a 5×5 matrix B with $BA=1$.

Let $A=\begin{pmatrix} 0&0&0&0&0\\ 0&1&1&1&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{pmatrix}$. Then A is not the zero matrix.

If $B=\begin{pmatrix} B_{11}&B_{12}&B_{13}&B_{14}&B_{15}\\ B_{21}&B_{22}&\cdots&&\\ &&\ddots&\vdots&\\ B_{51}&\cdots&\vdots&&B_{55}\end{pmatrix}$ is any 5×5 matrix

then $BA=\begin{pmatrix} 0&B_{12}&B_{13}&B_{14}&0\\ 0&B_{22}&B_{23}&B_{24}&0\\ 0&B_{32}&B_{33}&B_{34}&0\\ \vdots&\vdots&\vdots&&\vdots\\ 0&B_{52}&B_{53}&B_{54}&0\end{pmatrix}$ which is never the identity

(no matter what matrix B we chose)

since the $(1,1)$ entry is not 1. //

HW2 Problem 6 Prove: If A is an $n \times n$ matrix and A^{-1} exists then A^{-1} is unique.

Proof Assume A is an $n \times n$ matrix and A is invertible.

To show: The inverse of A is unique.

To show: If B_1 and B_2 are both inverses of A then $B_1 = B_2$.

Assume B_1 and B_2 are both inverses of A.

To show: $B_1 = B_2$.

We know $B_1 A = 1$ and $AB_1 = 1$, since B_1 is ~~an~~ inverse of A.

We know $B_2 A = 1$ and $AB_2 = 1$, since B_2 is an inverse of A.

So

$$B_1 = B_1 \cdot 1 = B_1(AB_2) = (B_1 A)B_2 = 1 \cdot B_2 = B_2.$$

So $B_1 = B_2$. //

<u>HW2 Problem 7</u> Prove: If A and B are $n\times n$ matrices and A^{-1} and B^{-1} exist then $(AB)^{-1}$ exists.

<u>Proof</u> Assume A and B are invertible matrices

To show: AB is invertible.

To show: There exists a matrix C such that

$C\cdot(AB)=1$ and $(AB)C=1$.

We know A^{-1} exists and $AA^{-1}=1$ and $A^{-1}A=1$.

We know B^{-1} exists and $BB^{-1}=1$ and $B^{-1}B=1$.

Let $C=B^{-1}A^{-1}$.

To show (a) $C\cdot(AB)=1$

(b) $(AB)\cdot C=1$.

(a) $C\cdot(AB)=B^{-1}A^{-1}\cdot AB=B^{-1}(A^{-1}A)B$

$=B^{-1}\cdot 1\cdot B=B^{-1}B=1.$

(b) $(AB)\cdot C=(AB)(B^{-1}A^{-1})=A(BB^{-1})A^{-1}$

$=A\cdot 1\cdot A^{-1}=AA^{-1}=1.$

So $B^{-1}A^{-1}$ is the inverse of AB.

<u>HW2 Problem 8</u> Let A be an $n \times n$ matrix and assume that A^{-1} exists. Show that $(A^t)^{-1} = (A^{-1})^t$.

<u>Proof</u> Assume A is an invertible matrix.

To show: The inverse of (A^t) is $(A^{-1})^t$.

To show: If $C = (A^{-1})^t$ then C is the inverse of A^t.

Assume $C = (A^{-1})^t$.

To show: (a) $CA^t = 1$

(b) $A^t C = 1$.

(a) $C \cdot A^t = (A^{-1})^t A^t = (AA^{-1})^t$,

since $(AB)^t = B^t A^t$ from HW1 Problem 17.

So $C \cdot A^t = (AA^{-1})^t = 1^t = 1$.

(b) $A^t C = A^t (A^{-1})^t = (A^{-1}A)^t$,

since $(AB)^t = B^t A^t$ from HW1 Problem 17.

So $A^t C = (A^{-1}A)^t = 1^t = 1$ //.