MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

SAMPLE FINAL EXAM
December 20, 2006

This is a 2 hour exam. No books, notes or calculators are allowed. There are 16 problems on this exam. All problems are worth 10 points each. Doing the easier ones first will probably help to maximize your total points.

Name: ______________________________

TA and Section: ______________________________

Problem	Score
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
16.	
Total	

Problem 1. Verify the identity $\cosh(-x) = \cosh x$.

Problem 2. Explain why $\frac{dx^n}{dx} = nx^{n-1}$, for all positive integers n.

Problem 3. Differentiate $7x^5 - 11x^2$ with respect to $7x^2 - 15x$.

Problem 4. Find $\dfrac{dy}{dx}$ when $y = \cot^{-1}\left(\dfrac{1+\cos 3x}{1-\cos 3x}\right)^{1/2}$.

Problem 5. Calculate $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$ when $f(x) = e^{\cos x}$.

Problem 6. What is a critical point? Explain how to find critical points of a function $f(x)$?

Problem 7. Graph $y = f(x)$ when $x = 2y^2 - 1$. Also determine

(a) where $f(x)$ is defined,

(b) where $f(x)$ is continuous,

(c) where $f(x)$ is differentiable,

(d) where $f(x)$ is increasing and where it is decreasing,

(e) where $f(x)$ is concave up and where it is concave down,

(f) what the critical points of $f(x)$ are,

(g) where the points of inflection are, and

(h) what the asymptotes to $f(x)$ are (if $f(x)$ has asymptotes).

Problem 8. Find the local maxima and minima of $f(x) = (x-1)(x+2)^2$.

Problem 9. Compute $\displaystyle\int \frac{\sin 2x}{(a + b\cos 2x)^2}\, dx$.

Problem 10. Find the area bounded by the curve $y = x(x-3)(x-5)$, the x-axis and the lines $x = 0$ and $x = 5$.

Problem 11. Using integration find the volume generated by rotating the triangle with vertices at $(0,0)$, $(h,0)$, and (h,r) about the y-axis.

Problem 12. Explain why the average of the numbers $1, 1/2, 1/3, \ldots, 1/100$ is more than .04615 but less than .04705.

Problem 13. Evaluate $\lim_{x \to 0} (1 - 2x)^{1/x}$. You may use L'Hôpital's rule.

Problem 14. Find $\dfrac{dy}{dx}$ when $y = \sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$.

Problem 15. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15cm.

Problem 16. Explain why $\frac{dx^n}{dx} = nx^{n-1}$, for $n = 0$.