MATH 221, Lecture 8 September 22, 2000

Review for exam

We covered:

(1) Exponential function: e^x

$\frac{de^x}{dx} = e^x$, $\quad e^0 = 1$,

$e^{x+y} = e^x e^y$, $\quad e^{-x} = \frac{1}{e^x}$

$(e^x)^y = e^{xy}$ $\quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(2) The logarithm: $\ln x$

$\ln x$ is the function that undoes e^x.

$\ln 1 = 0$, $\quad \ln(ab) = \ln(a) + \ln(b)$,

$\ln\left(\frac{1}{a}\right) = -\ln a$, $\quad \ln(a^b) = b \ln a$

$\frac{d \ln x}{dx} = \frac{1}{x}$.

(3) Trig functions:

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$

$\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$

$\sec x = \frac{1}{\cos x}$ $\quad \csc x = \frac{1}{\sin x}$

$e^{ix} = \cos x + i \sin x$

$\sin^2 x + \cos^2 x = 1$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(-x) = -\sin x$, $\quad \cos(-x) = \cos x$

$\frac{d \sin x}{dx} = \cos x$, $\quad \frac{d \cos x}{dx} = -\sin x$

$\frac{d \tan x}{dx} = \sec^2 x$, $\quad \frac{d \cot x}{dx} = -\csc^2 x$

$\frac{d \sec x}{dx} = \tan x \sec x \qquad \frac{d \csc x}{dx} = -\cot x \csc x.$

(4) Inverse trig functions.

$\sin^{-1} x$ is the function that undoes $\sin x$

$\cos^{-1} x$ is the inverse function to $\cos x$

$\tan^{-1} x$ is the inverse function to $\tan x$

$\cot^{-1} x$ is the inverse function to $\cot x$

$\sec^{-1} x$ is the inverse function to $\sec x$

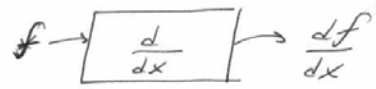
$\csc^{-1} x$ is the inverse function to $\csc x$.

$$\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d \cos^{-1} x}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d \tan^{-1} x}{dx} = \frac{1}{1+x^2}, \qquad \frac{d \cot^{-1} x}{dx} = \frac{-1}{1+x^2}$$

$$\frac{d \sec^{-1} x}{dx} = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d \csc^{-1} x}{dx} = \frac{-1}{x\sqrt{x^2-1}}.$$

(5) Derivatives:



(a) $\frac{dx}{dx} = 1$, (b) $\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$

(c) $\frac{d(cf)}{dx} = c\frac{df}{dx}$, if c is a constant,

(d) $\frac{d(fg)}{dx} = f\frac{dg}{dx} + \frac{df}{dx}g$, (e) $\frac{dx^n}{dx} = n x^{n-1}$,

(f) $\frac{dc}{dx} = 0$ if c is a constant,

(g) $\frac{d1}{dx} = 0$

(h) $\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$.