MATH 221, Lecture 40, December 15, 2000

Helpful techniques

(1) Multiplying out (1') Factoring

(2) Common denominator (2') Partial fractions

(3) Multiply top and bottom by the same thing (3') Add and subtract the same thing

(4) Completing the square (5) Change STUFF to $e^{\ln(\text{STUFF})}$

(6) Change $x = r\cos\theta$, $y = r\sin\theta$ to work with circles of radius r.

(7) Multiply by the conjugate

- (a) to divide complex numbers
- (b) to get rid of radicals added together
- (c) to deal with some integrals.

(8) Change messy trig functions to sines and cosines

(9) If its not how you want it, make it like you want it (in such a way that it is still equal to what it was before.

(10) Don't panic, just write one tiny step at a time.

Remarks for review.

(1) The word "prove" is the same as "explain why".
A problem that begins with the words
"Prove that" or "Show that" or "Explain why"
is exactly the same as a problem with the answer given.

(2) Unsimplifications for integrals:

$$\cos^2 x = \tfrac{1}{2}(\cos^2 x + \cos^2 x) = \tfrac{1}{2}(\cos^2 x + 1 - \sin^2 x) = \tfrac{1}{2}(1 + \cos 2x),$$

$$\sin^2 x = \tfrac{1}{2}(\sin^2 x + \sin^2 x) = \tfrac{1}{2}(1 - \cos^2 x + \sin^2 x) = \tfrac{1}{2}(1 - \cos 2x),$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} = \sec^2 x - 1,$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1-\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} = \csc^2 x - 1.$$

(3) The "theory" problems were (more or less) all done in class and so they could be called "regurgitation" problems. These are:
HW1 B1-11, HW2 A1-13, HW3 A1-31, HW3B1-6, HW3 C1-3, HW4 G1-9, HW6D1-8, HW7D1-3, H10E1-7, HW12B1-~~4~~5, HW12D 1-5.
These problems are the basis for the concepts in Math 221.

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Things to memorize for the exam for speed.

(1) Favourite derivatives:

$\frac{d\sin x}{dx} = \cos x$, $\frac{d\cos x}{dx} = -\sin x$, $\frac{d\tan x}{dx} = \sec^2 x$,

$\frac{d\csc x}{dx} = -\csc x\cot x$, $\frac{d\sec x}{dx} = \sec x\tan x$, $\frac{d\cot x}{dx} = -\csc^2 x$,

$\frac{de^x}{dx} = e^x$, $\frac{d\ln x}{dx} = \frac{1}{x}$

$\frac{d\sin^{-1}x}{dx} = \frac{1}{\sqrt{1-x^2}}$, $\frac{d\sec^{-1}x}{dx} = \frac{1}{x\sqrt{x^2-1}}$, $\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$

$\frac{d\cos^{-1}x}{dx} = \frac{-1}{\sqrt{1-x^2}}$, $\frac{d\csc^{-1}x}{dx} = \frac{-1}{x\sqrt{x^2-1}}$, $\frac{d\cot^{-1}x}{dx} = \frac{-1}{1+x^2}$

(2) Favourite limits:

$\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$

$\lim_{x\to 0} \frac{e^x-1}{x} = 1$, $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$.

(3) Favourite trig identities:

$\sin^2 x + \cos^2 x = 1$, $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$, $\sin 2x = 2\sin x\cos x$,

$\cos(x+y) = \cos x\cos y - \sin x\sin y$, $\cos 2x = \cos^2 x - \sin^2 x$

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(3) Favorite series:

$$\frac{1}{1-x} = 1+x+x^2+x^3+x^4+x^5+\cdots$$

$$1+x+x^2+x^3+\cdots+x^n = \frac{x^{n+1}-1}{x-1}$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\cdots$$

$$\sin x = x - \frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\frac{x^{11}}{11!}+\cdots$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\frac{x^{10}}{10!}+\cdots$$

Things to learn FOREVER

(1) $\frac{de^x}{dx} = e^x$, $e^{x+y} = e^x e^y$, $e^0 = 1$, $(e^a)^b = e^{ba}$

(2) $e^{ix} = \cos x + i\sin x$

(3) $\ln(ab) = \ln(a)+\ln(b)$, $\ln(a/b) = \ln(a) - \ln(b)$

$\ln(a^b) = b\ln a$,

(4) Formula 1, Formula 2, Formula 3

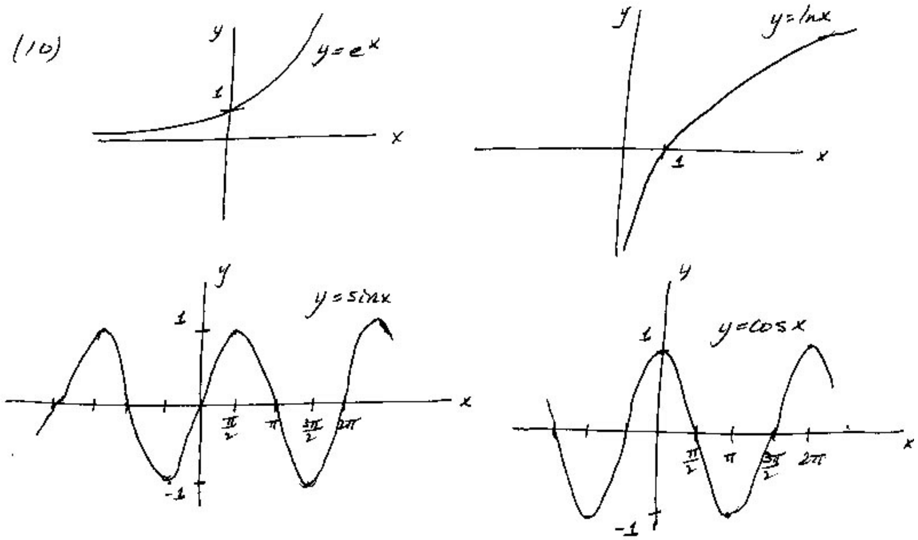
(5) The fundamental theorem of calculus,

(6) The chain rule

(7) The product rule

(8) 

(9) What π is (i.e. where it comes from)

(10) 

<u>Note</u>: The <u>major</u> <u>concepts</u> <u>of</u> <u>calculus</u> are

A Formula 3: $\left.\frac{df}{dx}\right|_{x=a} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$

says that $\left.\frac{df}{dx}\right|_{x=a}$ is a <u>rate</u>

and a <u>slope</u>

B Formula 1:

$$f(x) = f(a) + \left(\left.\frac{df}{dx}\right|_{x=a}\right)(x-a) + \left(\frac{\left.\frac{d^2f}{dx^2}\right|_{x=a}}{2!}\right)(x-a)^2 + \left(\frac{\left.\frac{d^3f}{dx^3}\right|_{x=a}}{3!}\right)(x-a)^3 + \left(\frac{\left.\frac{d^4f}{dx^4}\right|_{x=a}}{4!}\right)(x-a)^4 + \dots$$

says that

<u>You know $f(x)$ if you know its derivatives</u>, and

<u>You can use derivatives to find series</u>

C The fundamental theorem of calculus:
If $f(x)$ is differentiable between a and b,

$$\int_a^b f(x)\,dx = A(b) - A(a)$$

where $\int f(x)\,dx = A(x) + c$,

says that

<u>You can add up lots of little things</u>

<u>by undoing derivatives</u>.