MATH 221 Lecture 16, October 13, 2000 ①

Example Graph $f(x) = \dfrac{x^2-1}{x^3-4x}$

Notes

(a) $y = \dfrac{x^2-1}{x^3-4x} = \dfrac{(x+1)(x-1)}{x(x^2-4)} = \dfrac{(x+1)(x-1)}{x(x+2)(x-2)}$.

(b) If $x=1$ then $y=0$.

(c) If $x=-1$ then $y=0$.

(d) If $x \to 2^+$ then $y \to \infty$ $\left(\dfrac{\text{pos}\cdot\text{pos}}{\text{pos}\cdot\text{pos}\cdot\text{pos}}\right)$

(e) If $x \to 2^-$ then $y \to -\infty$ $\left(\dfrac{\text{pos}\cdot\text{pos}}{\text{pos}\cdot\text{pos}\cdot\text{neg}}\right)$

(f) If $x \to 0^+$ then $y \to \infty$ $\left(\dfrac{\text{pos}\cdot\text{neg}}{\text{pos}\cdot\text{pos}\cdot\text{neg}}\right)$

(g) If $x \to 0^-$ then $y \to -\infty$ $\left(\dfrac{\text{pos}\cdot\text{neg}}{\text{neg}\cdot\text{pos}\cdot\text{neg}}\right)$

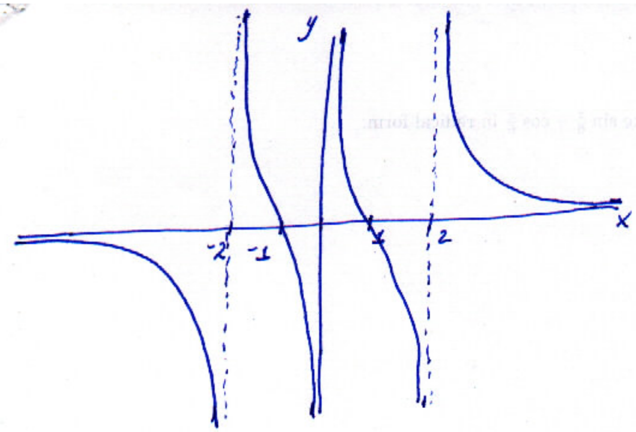
(h) If $x \to -2^+$ then $y \to \infty$ $\left(\dfrac{\text{neg}\cdot\text{neg}}{\text{neg}\cdot\text{pos}\cdot\text{neg}}\right)$

(i) If $x \to -2^-$ then $y \to -\infty$ $\left(\dfrac{\text{neg neg}}{\text{neg}\cdot\text{neg}\cdot\text{neg}}\right)$

(j) $y = \dfrac{x^2-1}{x^3-4x}$ is the same as $(-y) = \dfrac{(-x)^2-1}{(-x)^3-4(-x)}$

so if we flip y to $-y$ and x to $-x$ the graph stays the same.

②



(k) As $x \to \infty$ then $y \to 0^+$.

(l) As $x \to -\infty$ then $y \to 0^-$

Example Graph $\sqrt{x} + \sqrt{y} = 1$.

Notes:

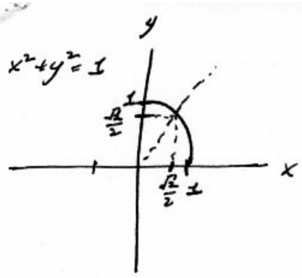
(a) If we switch x and y this graph stays the same

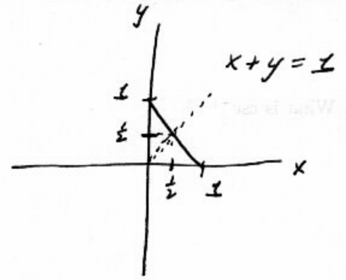
(b) If $x=0$ then $\sqrt{y}=1$, so $y=1^2=1$

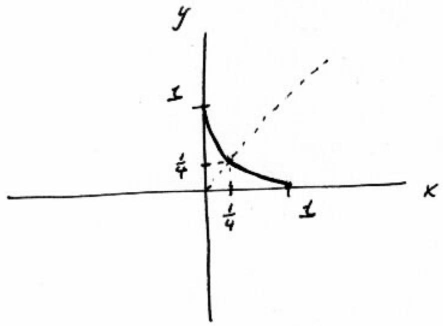
(c) If $y=0$ then $x=1$.

(d) If $x=y$ then $\sqrt{x}+\sqrt{x}=1$ and $\sqrt{x}=\frac{1}{2}$, $x=\frac{1}{4}$.

(e) This graph should be similar to $x^2+y^2=1$ or $x+y=1$



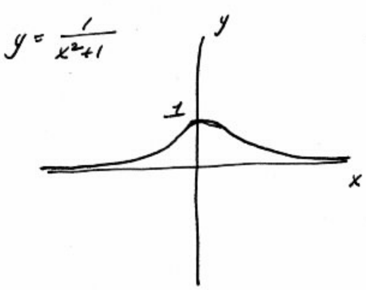




<u>Example</u> Graph $\frac{x^2-1}{x^2+1} = f(x)$.

<u>Notes</u>:

(a) $y = \frac{x^2-1}{x^2+1} = \frac{x^2+1-2}{x^2+1} = 1 - \frac{2}{x^2+1}$



<u>Notes</u>:

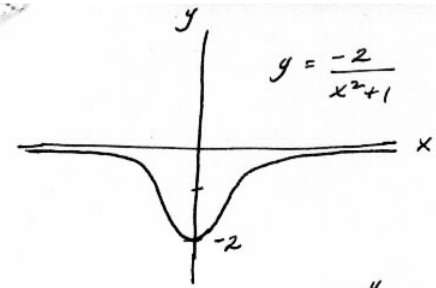
(a) If $x = 0$, $y = \frac{1}{0^2+1} = \frac{1}{1} = 1$

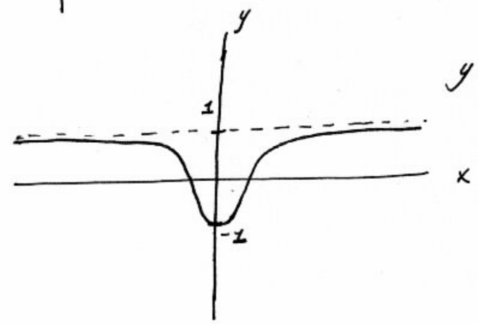
(b) If $x \to \infty$ then $y \to 0^+$

(c) If $x \to -\infty$ then $y \to 0^+$

(d) This graph stays the same if we flip x to $-x$,

$$y = \frac{1}{x^2+1} = \frac{1}{(-x)^2+1}$$





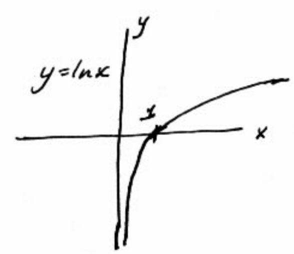
$y = 1 - \frac{2}{x^2+1} = \frac{x^2-1}{x^2+1}$

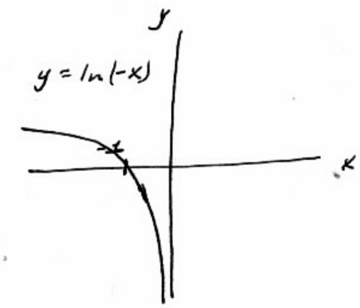
<u>Example</u> Graph $\ln(4-x^2) = f(x)$.

<u>Notes</u>:

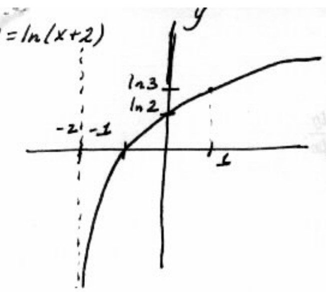
(a) $y = \ln(4-x^2) = \ln((2+x)(2-x)) = \ln(2+x) + \ln(2-x)$.

(b) If we flip x to $-x$ the graph stays the same since $y = \ln(4-x^2) = \ln(4-(-x)^2)$.

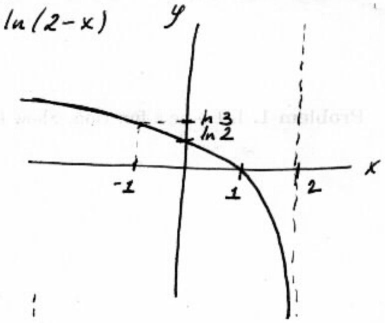


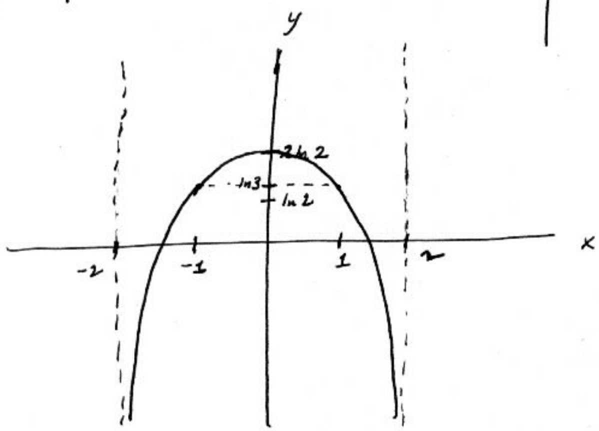


$y = \ln(x+2)$



$y = \ln(2-x)$





$y = \ln(4-x^2) = \ln(2+x) + \ln(2-x)$.

<u>Example</u> Graph $y = x^{2/3}(6-x)^{1/3}$

<u>Notes</u>:

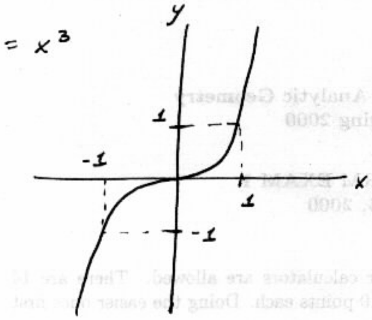
(a) If $x=0$ then $y=0$.

(b) If $x=6$ then $y=0$.

(c) If $x \to \infty$ then $y \to x^{2/3}(-x)^{1/3} = -x$.

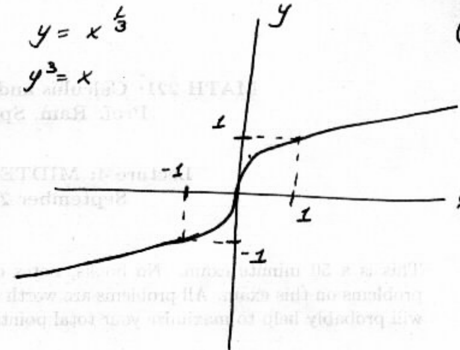
(d) If $x \to -\infty$ then $y \to \infty$ ($y \to -x$ again).

$y = x^3$

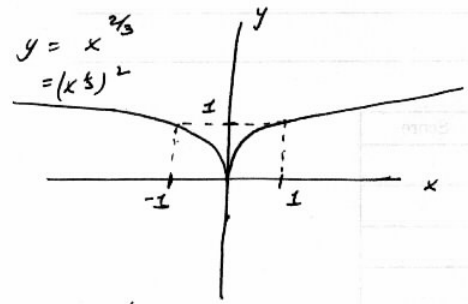


$y = x^{\frac{1}{3}}$

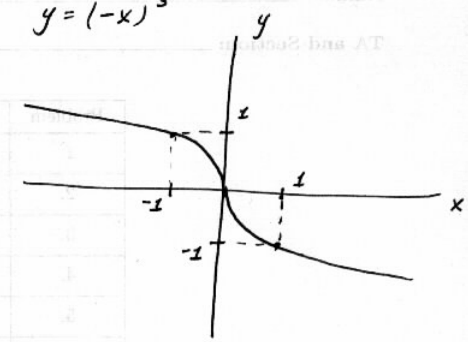
$y^3 = x$



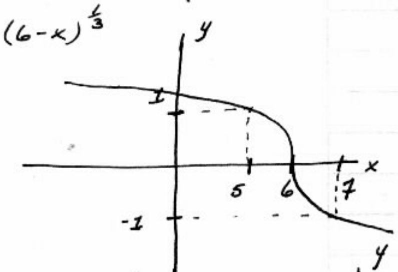
$y = x^{2/3}$
$= (x^{1/3})^2$

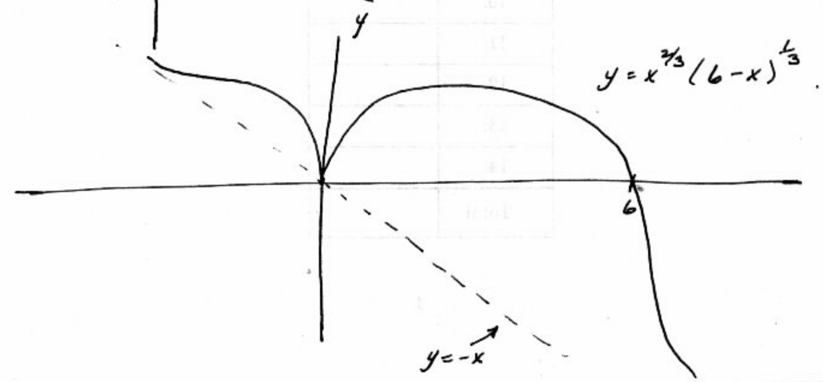


$y = (-x)^{\frac{1}{3}}$



$y = (6-x)^{\frac{1}{3}}$





$y = x^{2/3}(6-x)^{\frac{1}{3}}$.