MATH 221 Lecture 14, October 9, 2000 (1)

Graphing Techniques

(a) Basic Graphs

(b) Shifting

(c) Scaling

(d) Flipping

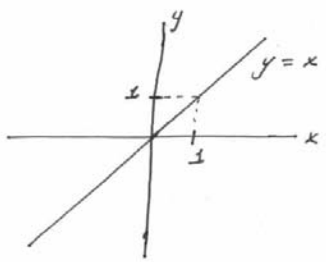
(e) limits

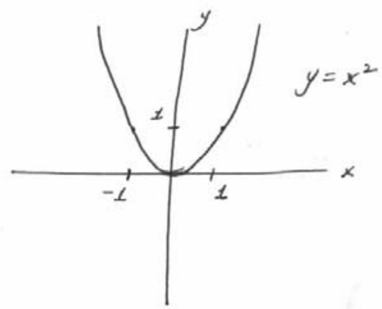
(f) asymptotes

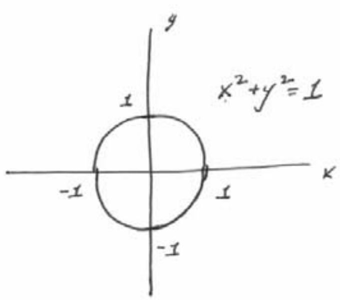
(g) Slopes: Increasing / Decreasing

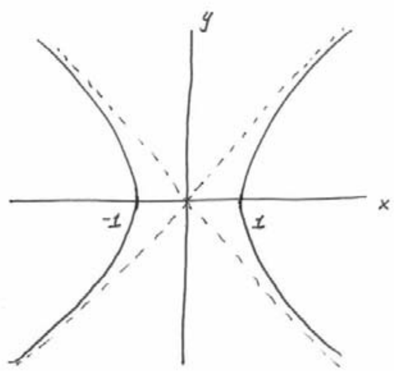
(h) Concave Up / Concave down points of Inflection.

Basic Graphs



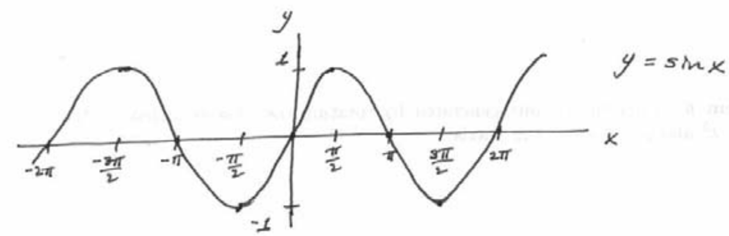


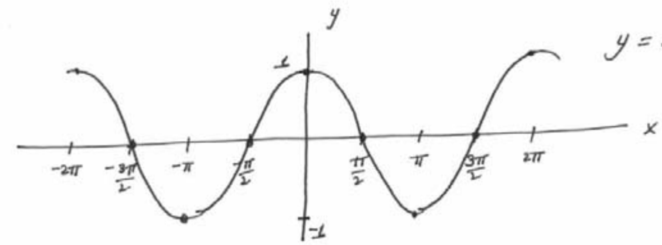


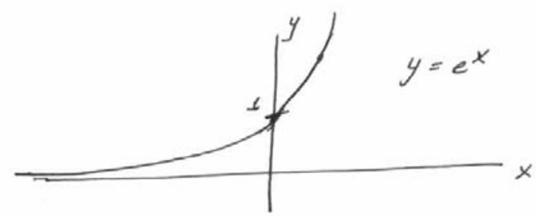


$x^2 - y^2 = 1$

(2)

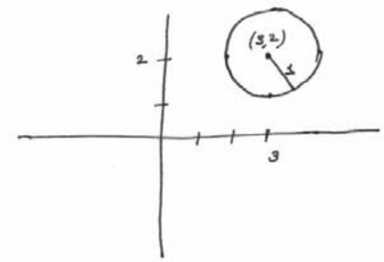






Shifting

Example: Graph $(x-3)^2 + (y-2)^2 = 1$



Notes:

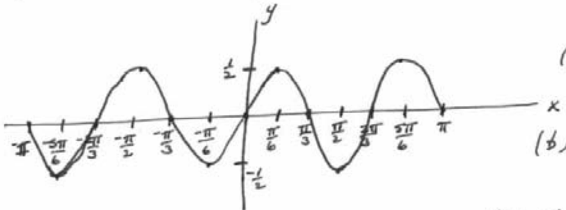
(a) $x^2+y^2=1$ is a basic circle of radius 1

(b) Center is shifted by 3 to the right in the x-direction

2 upwards in the y-direction.

Scaling

Example Graph $2y = \sin 3x$



Notes:

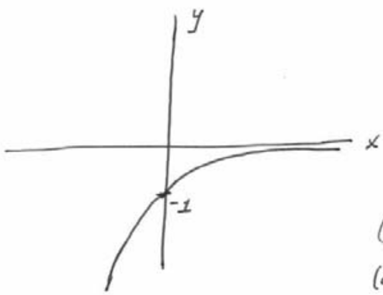
(a) $y = \sin x$ is the basic graph

(b) The x-axis is scaled (squished) by 3

(c) The y-axis is scaled by 2.

Flipping

Example Graph $y = -e^{-x}$.



Notes:

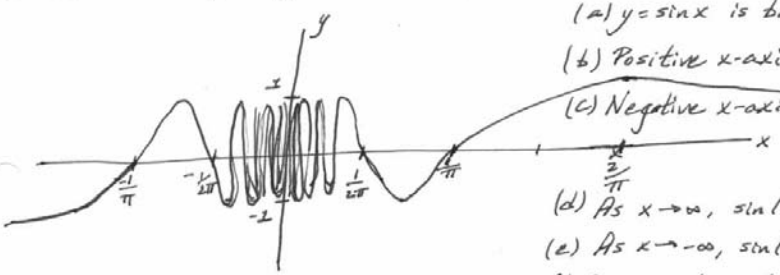
(a) $y = e^x$ is the basic graph.

(b) $y = -e^{-x}$ is the same as $-y = e^{-x}$

(c) The x-axis is flipped

(d) The y-axis is flipped.

Example Graph $y = \sin(\frac{1}{x})$



Notes:

(a) $y = \sin x$ is basic graph

(b) Positive x-axis is flipped

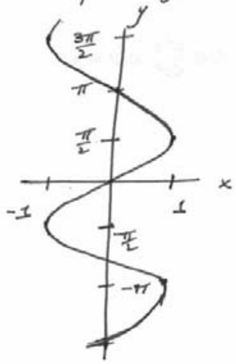
(c) Negative x-axis is flipped.

(d) As $x \to \infty$, $\sin(\frac{1}{x}) \to 0^+$

(e) As $x \to -\infty$, $\sin(\frac{1}{x}) \to 0^-$

(f) As $x \to 0^+$, $\sin(\frac{1}{x})$ goes between $+1$ and -1.

Example Graph $y = \sin^{-1} x$.



Notes:

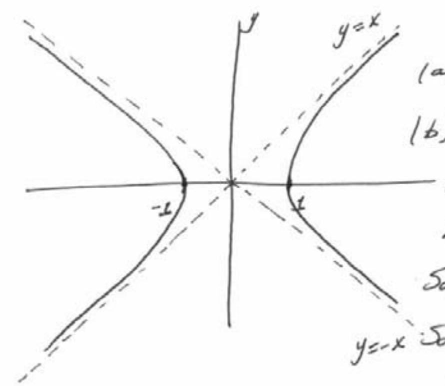
(a) $y = \sin x$ is basic graph.

(b) $y = \sin^{-1} x$ is same as $\sin y = x$ so x and y axis are switched from $y = \sin x$ graph.

Asymptotes

An asymptote of a graph $y = f(x)$ as $x \to a$ is another graph $y = g(x)$ that the original graph gets closer and closer to as x gets closer and closer to a.

Example Graph $x^2 - y^2 = 1$.



Notes:

(a) If $y = 0$ then $x = \pm 1$.

(b) $x^2 - y^2 = 1$ is the same as $1 - \frac{y^2}{x^2} = \frac{1}{x^2}$.

As $x \to \infty$ this becomes $1 - \left(\frac{y}{x}\right)^2 = 0$.

So, as $x \to \infty$ $y^2 = x^2$. So $y = \pm x$

So $y = x$ is an asymptote as $x \to \infty$

$y = -x$ is an asymptote as $x \to \infty$.

As $x \to \infty$, $1-\frac{y^2}{x^2}=\frac{1}{x^2}$ becomes $1-\left(\frac{y}{x}\right)^2=0$.

So, as $x \to \infty$ the graph is $y^2=x^2$, or $y=\pm x$.

So $y=x$ is an asymptote as $x \to -\infty$.

$y=-x$ is also an asymptote as $x \to -\infty$.

Example $f(x)=\begin{cases}\dfrac{1-\cos x}{x^2}, & \text{if } x \neq 0, \\ 1, & \text{if } x=0\end{cases}$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\lim_{x\to 0}\frac{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\right)}{x^2}$$

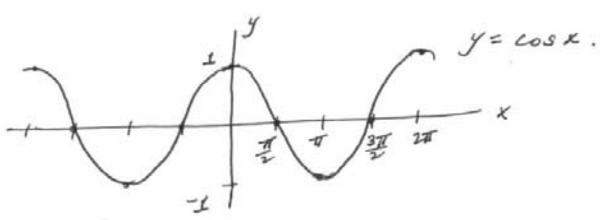
$$=\lim_{x\to 0}\frac{\frac{x^2}{2!}-\frac{x^4}{4!}+\frac{x^6}{6!}-\cdots}{x^2}=\lim_{x\to 0}\frac{1}{2}-\frac{x^2}{4!}+\frac{x^4}{6!}-\frac{x^6}{8!}+\cdots$$

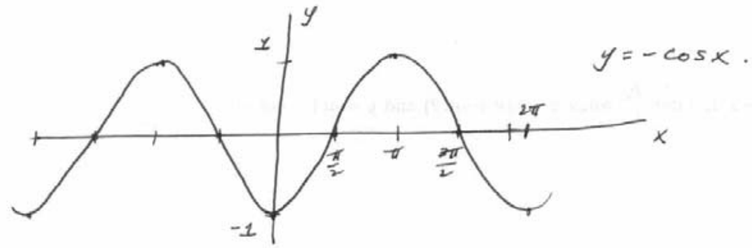
$$=\frac{1}{2}-0+0-0+\cdots=\frac{1}{2}.$$

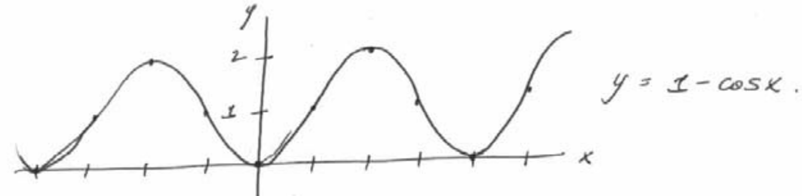
So $\lim_{x\to 0} f(x)=\frac{1}{2}$. Since $f(0)=1$, $\lim_{x\to 0} f(x)\neq f(0)$.

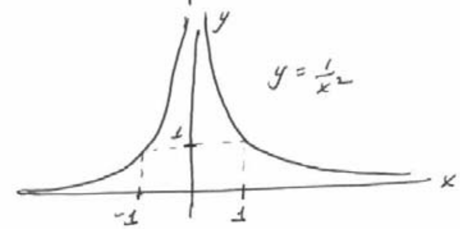
So $f(x)$ is not continuous at $x=0$.

$y=\cos x$

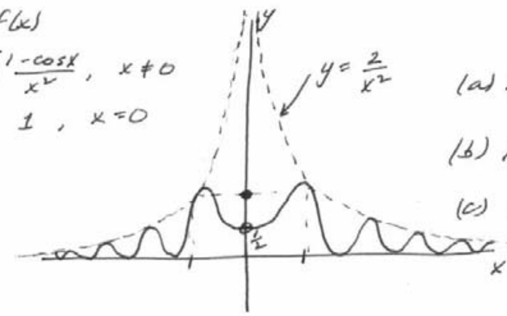








$y=f(x)=\begin{cases}\dfrac{1-\cos x}{x^2}, & x\neq 0 \\ 1, & x=0\end{cases}$



Notes:

(a) As $x\to 0$, $\frac{1-\cos x}{x^2}\to\frac{1}{2}$

(b) As $f(0)=1$

(c) At the peaks of $1-\cos x$

$$\frac{1-\cos x}{x^2}=\frac{2}{x^2}.$$