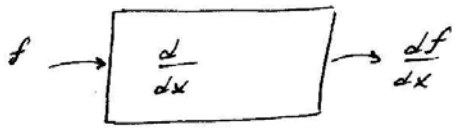
MATH 221 Lecture 1 September 6, 2000

Calculus is the study of

(1) Derivatives (3) Applications of Derivatives

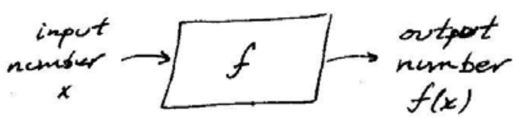
(2) Integrals (4) Applications of Integrals

A derivative is a creature you put a function into, it chews on it and spits out a different function



The integral is the derivative backwards:

A function is one down on the food chain

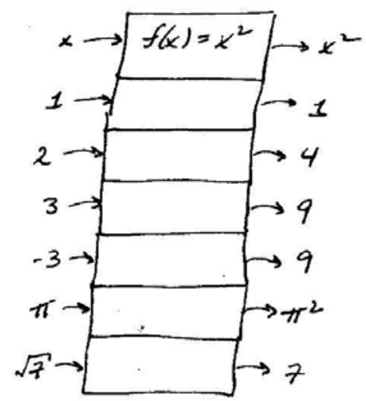


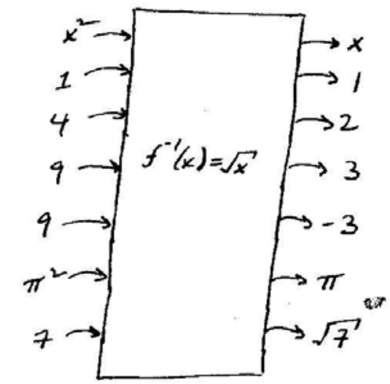
Functions take a number as input, chew on it a bit and spit out a number.

The inverse function to f is f backwards

Example

The inverse function is





The inverse function is not always a function because there might be some uncertainty about what the inverse function will spit out

Numbers are at the very bottom of the food chain.

Numbers

At some point humankind wanted to count things and discovered the positive integers

$1, 2, 3, 4, 5, 6, \ldots$

Great for counting something

BUT what if you don't have anything i.e. nothing, nulla, zip

and so we discovered the nonnegative integers

$0, 1, 2, 3, 4, 5, \ldots$

GREAT for adding $1+3=4$, $5+0=5$, $9+16=25$

BUT not so great for subtracting $1-3=???$

and so we discovered the integers

$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$

GREAT for adding, subtracting and multiplying

$-2 \cdot 4 = -8$, $7 \cdot 6 = 42$, $(-7)(-6) = 42$

BUT not so great if you only want part of the sausage ...

... and so we discovered the rational numbers

$\frac{a}{b}$, a an integer, b an integer, $b \neq 0$.

GREAT for addition subtraction, multiplication and division,

BUT not so great for finding $\sqrt{2}$ ...

... and so we discovered the real numbers

all finite and infinite decimal expansions.

Examples: $\sqrt{2} = 1.414\ldots$

$e = 2.71828\ldots$

$\pi = 3.1415926\ldots$

$\frac{1}{6} = .16666\ldots$

$\frac{1}{8} = .125 = .1250000\ldots$

GREAT for addition subtraction, multiplication, and division

BUT not so great for finding $\sqrt{-9}$ ...

and so we discovered the complex numbers ⑤

$a+bi$, a a real number, b a real number, $i = \sqrt{-1}$.

Examples: $3+4i$ $\quad 0+10i = 10i$

$7+9i$ $\quad \pi + 0i = \pi$

$3.2+6.7i$ $\quad \frac{1}{3}+\frac{2}{6}i = \frac{1}{3}+\frac{1}{3}i$

$5+0i=5$ $\quad \sqrt{7}+\sqrt{2}i$

and $(3i)^2 = 3^2 i^2 = 9i^2 = -9$. So $\sqrt{-9} = 3i$

GREAT.

Addition: $(3+4i)+(7+9i) = 10+13i$

Subtraction: $(3+4i)-(7+9i) = 3-7+4i-9i = -4-5i$

Multiplication: $(3+4i)(7+9i) = 3(7+9i)+4i(7+9i)$

$$= 21+27i+28i+36i^2$$

$$= 21+27i+28i-36$$

$$= -15+55i$$

Division: $\dfrac{3+4i}{7+9i} = \dfrac{(3+4i)}{(7+9i)}\dfrac{(7-9i)}{(7-9i)}$

⑥

$$= \frac{21-27i+28i+36}{49-63i+63i+81} = \frac{57+i}{130}$$

$$= \frac{57}{130} + \frac{1}{130}i$$

Square roots: $\sqrt{-3+4i} = \pm(1+2i)$

since $(1+2i)^2 = 1+2i+2i+4i^2 = 1+4i-4 = -3+4i$

and $(-(1+2i))^2 = (1+2i)^2 = -3+4i$

Another way is: $\sqrt{-3+4i} = a+bi$

So $-3+4i = (a+bi)^2 = a^2 + abi + abi + bi^2$

$$= a^2 - b^2 + 2abi$$

So $a^2-b^2 = -3$ and $2ab = 4$.

Solve for a and b.

$b = \frac{4}{2a} = \frac{2}{a}$. So $a^2 - \left(\frac{2}{a}\right)^2 = -3$

So $a^2 - \frac{4}{a^2} = -3$

So $a^4 - 4 = -3a^2$

So $a^4 + 3a^2 - 4 = 0$

So $(a^2+4)(a^2-1) = 0$

So $a^2 = -4$ or $a^2 = 1$.

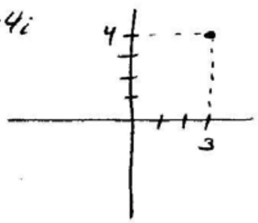
So $a = \pm 1$. So $b = \frac{2}{\pm 1} = 2$ or -2.

So $a+bi = 1+2i$ or $a+bi = -1-2i$.

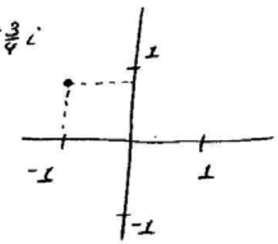
So $\sqrt{-3+4i} = \pm(1+2i)$.

Graphing

$3+4i$



$-1+\frac{3}{4}i$



Factoring $x^2+5 = (x+\sqrt{5}\,i)(x-\sqrt{5}\,i)$

$$(x^2+x+1) = \left(x-\left(\frac{-1+\sqrt{-3}}{2}\right)\right)\left(x-\left(\frac{-1-\sqrt{3}}{2}\right)\right)$$

$$= \left(x-\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)\right)\left(x-\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)\right)$$

The fundamental theorem of algebra is one reason why the complex number system is "the right" number system to use. It says that any polynomial can be factored completely as

$(x-u_1)(x-u_2)(x-u_3)\cdots(x-u_n)$ where $u_1, u_2, \ldots, u_n$ are some complex numbers.