HOMEWORK 1
DUE September 11, 2006

Problem A. Numbers

(1) What are the positive integers and why do we care?

(2) What are the nonnegative integers and why do we care?

(3) What are the rational numbers and why do we care?

(4) What are the real numbers and why do we care?

(5) What are the complex numbers and why do we care?

(6) What do $2 + 3$, $2 + \frac{5}{7}$, $\frac{4}{9} + \frac{5}{7}$, $2 + 1.4$ and $2 + \sqrt{2}$ mean?

(7) What do x^2, $\frac{1}{x}$ and $\sqrt{x}$ mean?

(8) What do $a + b$, $a + \frac{b}{c}$, $\frac{a}{b} + \frac{c}{d}$ mean?

(9) What do 2^3, $2^{\frac{5}{7}}$, $\left(\frac{2}{3}\right)^{\frac{5}{7}}$, 2^x, $2^{1.4}$ and $2^{\sqrt{2}}$ mean?

(10) What do a^b, $a^{\frac{b}{c}}$, $\left(\frac{a}{b}\right)^{\frac{c}{d}}$, 2^x, and x^2 mean?

(11) What do x^x and $x^{\sqrt{x}}$ mean?

Problem B. Computing with complex numbers

(1) Find a complex number z such that $z + w = w$ for all other complex numbers w.

(2) Find a complex number x such that $xw = w$ for all other complex numbers w.

(3) Compute $(3 - 7i) + (2 + 5i)$ and graph the result.

(4) Compute $(-12 + 3i) - (7 - 5i)$ and graph the result.

(5) Compute $(4 + 8i)(2 - 3i)$ and graph the result.

(6) Compute $\dfrac{-15 + i}{4 + 2i}$ and graph the result.

(7) Compute $(3 - 2i)^3$ and graph the result.

(8) Compute $\sqrt{2i}$ and graph the result.

(9) Compute $\dfrac{1}{a + bi}$ and graph the result, where $a, b \in \mathbb{R}$.

(10) Compute $(3 - 5i) + (7 + 2i)$ and graph the result.

(11) Compute $(5 - 2i) - (3 - 6i)$ and graph the result.

(12) Compute $(2 - 4i)(3 + 2i)$ and graph the result.

(13) Compute $\dfrac{6 - i}{4 + 2i}$ and graph the result.

(14) Compute $1^{1/4}$ and graph the result.

(15) Compute $16^{1/4}$ and graph the result.

(16) Compute $(27^{1/3})^4$ and $27^{(4+1/3)}$ graph the result.

(17) Compute $1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{32} + \dfrac{1}{64} + \cdots$.

(18) Compute $1 \cdot 2$, $1 \cdot 2 \cdot 3$, $1 \cdot 2 \cdot 3 \cdot 4$, $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ and $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6$.

(19) Compute $1 + \dfrac{1}{1} + \dfrac{1}{1 \cdot 2} + \dfrac{1}{1 \cdot 2 \cdot 3} + \dfrac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \dfrac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \cdots$.

Problem C. Functions

(1) What is x^2?

(2) What is e^x?

(3) What is $\sin x$?

(4) What is $\cos x$?

(5) What is $\tan x$?

(6) What is $\cot x$?

(7) What is $\sec x$?

(8) What is $\csc x$?

(9) What is $\sinh x$?

(10) What is $\cosh x$?

(11) What is $\tanh x$?

(12) What is $\coth x$?

(13) What is $\operatorname{sech} x$?

(14) What is $\operatorname{csch} x$?

(15) What is $\sqrt{x}$?

(16) What is $\ln x$?

(17) What is $\sin^{-1} x$?

(18) What is $\cos^{-1} x$?

(19) What is $\tan^{-1} x$?

(20) What is $\cot^{-1} x$?

(21) What is $\sec^{-1} x$?

(22) What is $\csc^{-1} x$?

(23) What is $\sinh^{-1} x$?

(24) What is $\cosh^{-1} x$?

(25) What is $\tanh^{-1} x$?

(26) What is $\coth^{-1} x$?

(27) What is $\operatorname{sech}^{-1} x$?

(28) What is $\operatorname{csch}^{-1} x$?

Problem D. Function identities

(1) Explain why $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$.

(2) Explain why $\dfrac{x^n - 1}{x - 1} = 1 + x + x^2 + x^3 + \cdots + x^{n-1}$.

(3) Find all possibilities for $c_0, c_1, c_2 \ldots$ so that $f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$ satisfies $f(x + y) = f(x)f(y)$.

(4) Explain why $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \dfrac{x^5}{5!} + \dfrac{x^6}{6!} + \cdots$.

(5) Explain why $\ln x$ is the inverse function to e^x.

(6) Verify the identity $e^{x+y} = e^x e^y$.

(7) Verify the identity $e^{-x} = \dfrac{1}{e^x}$.

(8) Verify the identity $(e^x)^n = e^{nx}$.

(9) Verify the identity $e^0 = 1$.

(10) Verify the identity $\ln(xy) = \ln x + \ln y$.

(11) Verify the identity $-\ln x = \ln(1/x)$.

(12) Verify the identity $\ln x^n = n \ln x$.

(13) Verify the identity $\ln 1 = 0$.

(14) Explain why $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$.

(15) Explain why $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$.

(16) Verify the identity $e^{ix} = \cos x + i \sin x$.

(17) Verify the identity $\cos^2 x + \sin^2 x = 1$.

(18) Verify the identity $\sin(-x) = -\sin x$.

(19) Verify the identity $\cos(-x) = \cos x$.

(20) Verify the identity $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

(21) Verify the identity $\cos(x + y) = \cos x \cos y - \sin x \sin y$.

(22) Verify the identity $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$.

(23) Verify the identity $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$.

(24) Explain why $\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \cdots$.

(25) Explain why $\sin x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} + \cdots$.

(26) Verify the identity $e^{ix} = \cosh x + \sinh x$.

(27) Verify the identity $\cosh^2 x - \sinh^2 x = 1$.

(28) Verify the identity $\sinh(-x) = -\sinh x$.

(29) Verify the identity $\cosh(-x) = \cosh x$.

(30) Verify the identity $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

(31) Verify the identity $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$.

(32) Verify the identity $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

(33) Verify the identity $\sinh x = \dfrac{e^x - e^{-x}}{2}$.

Problem E. Trigonometric function identities

(1) Verify the identity $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$.

(2) Verify the identity $\sin(x/2) = \pm\sqrt{\dfrac{1 - \cos x}{2}}$.

(3) Verify the identity $\cos 3x = \cos^3 x - 3 \cos x \sin^2 x$.

(4) Verify the identity $\sin 3x = 3 \cos^2 x \sin x - \sin^3 x$.

(5) Verify the identity $\sin^2 A \cot^2 A = (1 - \sin A)(1 + \sin A)$.

(6) Verify the identity $\tan B = \dfrac{\cos B}{\sin B \cot^2 B}$.

(7) Verify the identity $\dfrac{\tan V \cos V}{\sin V} = 1$.

(8) Verify the identity $\sin E \cot E + \cos E \tan E = \sin E + \cos E$.

(9) Verify the identity $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x} - 1 = 0$.