

**MATH 221: Calculus and Analytic Geometry**  
**Prof. Ram, Fall 2006**

**HOMEWORK 3: SELECTED ANSWERS**

**Problem C. Computing some derivatives**

- (1)  $\frac{dy}{dx} = 20x + 27.$
- (2)  $\frac{dy}{dx} = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} - \frac{3}{2}x^{-5/2}.$
- (3)  $\frac{dy}{dx} = (2x - 5)(3x - 4)^2(30x - 61).$
- (4)  $\frac{dy}{dx} = 2ex - \frac{3\pi}{x^4} + \frac{7}{2}x^{5/2}.$
- (5)  $\frac{dy}{dx} = -\frac{2(x - 3)}{(x - 4)^3}.$
- (6)  $\frac{dy}{dx} = \frac{3x^2 + 10x + 12}{(4 - x^2)^2}.$
- (7)  $\frac{dy}{dx} = \frac{1 - x}{(1 - 2x)^{3/2}}.$
- (8)  $\frac{dy}{dx} = \frac{1}{\sqrt{x}(1 - \sqrt{x})^2}.$
- (9)  $\frac{dy}{dx} = -\frac{1}{(x - 1)^2} - \frac{1}{(x + 3)^2}.$
- (10)  $\frac{dy}{dx} = \frac{a}{\sqrt{a^2 - x^2}(a + \sqrt{a^2 - x^2})}.$
- (11)  $\frac{dy}{dx} = \frac{x^2 + 2x + 2}{(x + 1)^2}.$
- (12)  $\frac{dy}{dx} = \frac{-3}{2\sqrt{x}(x - 3)^{3/2}}.$
- (13)  $\frac{dy}{dx} = \frac{-2nx^{n-1}}{(x^n - 1)^2}.$
- (14)  $\frac{dy}{dx} = \frac{2x}{(1 - x^2)(1 - x^4)^{1/2}}.$
- (15)  $\frac{dy}{dx} = \frac{4x^2 + 1}{x^2(x^2 + 1)^{3/2}}.$

$$(16) \frac{dy}{dx} = nu^{n-1} \frac{du}{dx}.$$

$$(17) \frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}.$$

#### Problem D. Correcting derivative identities

$$(1) \frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}.$$

$$(2) \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}.$$

$$(3) \frac{d}{dx}(u \cdot v) = u \frac{dv}{dx} + \frac{du}{dx} v.$$

#### Problem F. Derivatives at a point

$$(1) \left. \frac{dy}{dx} \right|_{x=3} = 1476.$$

$$(2) \left. \frac{dy}{dx} \right|_{x=3} = 9.$$

#### Problem G. Derivatives with respect to functions

$$(1) \frac{dy}{dx} = \frac{2t^4 + 8}{5t^7}.$$

$$(2) \frac{dy}{dx} = \frac{1}{(1+x^2)^2}.$$

$$(3) \frac{dy}{dx} = \frac{(ad - bc)(c_1x + d_1)^2}{(a_1d_1 - b_1c_1)(cx + d)^2}.$$

$$(4) \frac{dy}{dx} = \frac{3}{2}x.$$

$$(5) \frac{dy}{dx} = \frac{-1}{x^2(1 + \sqrt{1-x^4})}.$$

$$(6) \frac{dy}{dx} = \frac{1-x^2}{3x^2(1+x^2)^2}.$$

$$(7) \frac{dy}{dx} = \frac{x + \sqrt{1 - x^2}}{-x}.$$

$$(8) \frac{dy}{dx} = \frac{35x^4 - 22x}{14x - 15}.$$

### Problem H. Derivatives of parametric equations

$$(1) \frac{dy}{dx} = \frac{-1}{t^2}.$$

$$(2) \frac{dy}{dx} = 1/t.$$

$$(3) \frac{dy}{dx} = -1.$$

$$(4) \frac{dy}{dx} = \frac{b(t^2 - 1)}{2at}.$$

$$(5) \frac{dy}{dx} = \frac{t^3 + 2t - t^{-1}}{2}.$$

$$(6) \frac{dy}{dx} = \frac{b(1 + t^2)}{2at}.$$

$$(7) \frac{dy}{dx} = \frac{t(2 - t^3)}{1 - 2t^3}.$$

$$(8) \frac{dy}{dx} = -x/y.$$

### Problem I. Implicit differentiation

$$(1) \frac{dy}{dx} = \frac{x(2a^2y^2 - x^2)}{y(y^2 - 2a^2x^2)}.$$

$$(2) \frac{dy}{dx} = \frac{-b^2x}{a^2y}.$$

$$(3) \frac{dy}{dx} = \frac{2axy^2 - x^4}{y^4 - 2ax^2y}.$$

$$(6) \frac{dy}{dx} = -\frac{ax + hy + g}{hx + by + f}.$$

**Problem J. Derivatives with trigonometric functions.**

- (1)  $\frac{dy}{dx} = 3 \cos(3x + 2).$
- (2)  $\frac{dy}{dx} = 2x^3(\sin x^4)^{-1/2} \cos x^4.$
- (3)  $\frac{dy}{dx} = x^2 \cos x + 2x \sin x.$
- (4)  $\frac{dy}{dx} = 2 \sin 2x.$
- (5)  $\frac{dy}{dx} = 2x \cos x^2 - \left( \frac{(1 + x^2) \sec^2 x - 2x \tan x}{(1 + x^2)^2} \right).$
- (6)  $\frac{dy}{dx} = - \left( \frac{2x \sin x + 4 \sin x + 2 \cos x + 2}{(x + 2)^2} \right).$
- (7)  $\frac{dy}{dx} = 2x + \frac{\sin x - x \cos x}{\sin^2 x}.$
- (8)  $\frac{dy}{dx} = 2 \cos x.$
- (9)  $\frac{dy}{dx} = \frac{1}{6} \sec(x/3) \tan(x/3).$
- (10)  $\frac{dy}{dx} = (\cos x - \sin x) \cos(\sin x + \cos x).$
- (11)  $\frac{dy}{dx} = -2 \csc 2x \cos 2x.$
- (12)  $\frac{dy}{dx} = 2x \left( \cot x + \frac{\tan x}{1 + x^2} \right) + \frac{x^2 - 1}{(1 + x^2)^2} ((1 + x^2) \sec^2 x - (1 + x^2)^2 \csc^2 x - 2x \tan x).$
- (13)  $\frac{dy}{dx} = \frac{-\frac{d\theta}{dx}}{\sqrt{\cos 2\theta}(\cos \theta + \sin \theta)}.$
- (14)  $\frac{dy}{dx} = \frac{2 \cos x}{(1 - \sin x)^2}.$
- (15)  $\frac{dy}{dx} = \frac{1}{2} \sec^2(x/2).$
- (16)  $\frac{dy}{dx} = x^3 \tan(x/2) \sec^2(x/2) + 3x^2 \tan^2(x/2).$
- (17)  $\frac{dy}{dx} = \frac{-\sec^2(\cos(\sin \theta)) \sin(\sin \theta) \cos \theta \cdot d\theta}{dx}.$

**Problem K. Derivatives with exponentials and logs.**

$$(1) \frac{dy}{dx} = 2ex - \frac{3\pi}{x^4} + \frac{7}{2}x^{5/2}.$$

$$(2) \frac{dy}{dx} = a^{ax+b+1} \ln a.$$

$$(3) \frac{dy}{dx} = 3x^2 a^{x^3} \ln a.$$

$$(4) \frac{dy}{dx} = 2 \cdot 6^{2x} \ln 6.$$

$$(5) \frac{dy}{dx} = \frac{2ax}{ax^2 + b}.$$

$$(6) \frac{dy}{dx} = 3x^2.$$

$$(7) \frac{dy}{dx} = 2(e^{2x} + e^{-2x}).$$

$$(8) \frac{dy}{dx} = 2(x+1)e^{x^2+2x}.$$

$$(9) \frac{dy}{dx} = ax^{a-1}a^x + x^a a^x \ln a.$$

$$(10) \frac{dy}{dx} = (x+1)e^x.$$