MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

HOMEWORK 12: SELECTED ANSWERS

Problem A. Length of a plane curve.

(2) 10.5 (3) $6a$ (4) 12

(5) $(8/27)(10\sqrt{10}-1)$ (6) $14/3$

(7) $53/6$ (8) $123/32$ (9) $(4/27)(10\sqrt{10}-1)$

(10) $a\pi^2/8$ (11) 8 (12) 12 (13) $21/2$

(14) $27/20$ (15) $19/3$ (16) $f(x) = a \pm x\sqrt{A^2-1}$, $|A| \geq 1$

(17) No

Problem B. Surface area.

(2) $4\pi^2 r^2$ (3) $99\pi/2$ (4) $(\pi/27)(10\sqrt{10}-1)$

(5) $(\pi/6)(17\sqrt{17}-1)$ (6) $1823\pi/18$ (7) $253\pi/20$

(8) $(2\pi/3)(2\sqrt{2}-1)$ (9) $12\pi a^2/5$ (10) $(2\pi/3)(26\sqrt{26}-2\sqrt{2})$

(11) $56\pi\sqrt{3}/5$ (12) $424\pi/15$ (13) $153\pi/40$

Problem C. Center of mass.

(1) At the intersection of the lines through each vertex which are perpendicular to the opposite side.

(2) At $(0, (2/\pi)r, 0)$ if the center is at $(0,0)$ and the y-axis cuts the semicircle in half.

(3) At $(0, (8/15)r, 0)$ if the hemisphere is sitting on the x-z plane with its apex at $(0, r, 0)$.

(4) $(4a/3\pi, 4a/3\pi)$ (5) $(0, (2/5)h^2)$ (6) $(2a/3(4-\pi), 2a/3(4-\pi))$

(7) $(\pi/2, \pi/8)$ (8) $(2/5, 1)$ (9) $(3/7)h$ (10) $(3/5)h$

(11) On the axis of the cone $3h/4$ from the vertex.

(12) On the axis of the cone $3h/5$ from the vertex.

(13) At $(0, \pi r/4)$ if the semicircle is positioned as in (2).

(14) At $(0, (3/8)r, 0)$ if the hemisphere is positioned as in (3).

(15) At $(0, (1/2)r, 0)$ if the hemisphere is positioned as in (3).

(16) $(0, 2c^2/5)$ (17) $(16/105, 8/15)$ (18) $(0, 12/5)$

(19) $(1, -3/5)$ (20) $(3/5, 1)$

(21) On the axis of the cone $3h/4$ from the vertex.

(22) $(0, 8/3)$ (23) $(4/5, 0)$

(24) On the axis of the cone $2h/3$ from the vertex.

(25) $(-r, 3r/(2+\pi))$ (26) $(17\sqrt{17} - 1)/12$

(27) $(2r/\pi, 2r/\pi)$

Problem B. Average value of a function.

(2) $50\frac{1}{2}$ (3) 126 (4) 117

(5) 215369396307555776631 07.46 (10) $2/\pi$ (11) 0

(12) $\frac{1}{2}$ (13) $\frac{1}{2}$ (14) 49/12 (15) $\frac{1}{2}$

(16) $\alpha\left(\dfrac{a+b}{2}\right) + \beta$ (17a) 200 cases (17b) 1 dollar per day

(18) $\frac{a}{3}(3\sqrt{3} - 1)$ (19a) $\frac{2}{3}b^2$ (19b) $\frac{2}{3}b$

(20a) 72 (20b) $82\frac{2}{3}$ (21) $50 + 28/\pi$