Math 521 Lecture 3: Homework 2
Fall 2004, Professor Ram
Due September 20, 2004

1 Exercises

1. Let $\sim$ be an equivalence relation on a set S. Show that the equivalence classes partition S.
2. Let $\{S_\alpha\}$ be a partition of a set S. Show that the S_α are the equivalence classes of an equivalence relation.
3. For each of the 2^3 subsets of {symmetric, reflexive, transitive} give an example of a naturally occurring relation that satisfies only the conditions specified by the subset.
4. What are your favourite relations?
5. Draw some pictures of relations. Explain!
6. Give an example of a nonassociative operation.
7. Give an example of a noncommutative operation.
8. Define left inverse and right inverse and give a naturally occurring example of a ring R and an element of R that has a left inverse but not a right inverse.
9. Show that the identity of an operation is unique.
10. Let S be a set with an operation. Show that if x is an element of S that has an inverse then the inverse is unique.
11. Let S be a set with an operation. Show that if x is an element of S that has a left inverse and a right inverse then the left inverse and the right inverse must be equal.
12. Let x be a matrix. Show that x has an inverse if and only if x is square and $\det(x)$ is invertible. (Since the necessary definitions for this problem have not been covered in this class, they were in Math 340, be sure to state clearly all the relevant definitions!!)
13. Give an examples of
    (a) a monoid without identity,
    (b) a monoid that is not a group,
    (c) a group that is not abelian,
    (d) an abelian group that is not a ring,

(e) a ring without identity
(f) a ring that is not commutative,
(g) a commutative ring that is not an integral domain,
(h) a division ring that is not a field, and
(a) (i)] a field that is finite.

14. Why isn't $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ a group?
15. What are your favourite homomorphisms?
16. What are the bijective homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}$?
17. Show that $1/(1-x) \in \mathbb{Z}[[x]]$.
18. Determine the invertible elements of $\mathbb{F}[[x]]$.
19. Show that $\mathbb{F}(x) = \left\{x^k p(x) \mid k \in \mathbb{Z}, p(x) \in \mathbb{F}[[x]]\right\}$.
20. Define e^x. Why is this definition natural?
21. Define $\ln x$. Where does $\ln x$ live? Why is this definition natural?
22. Show that $e^{(x+y)} = e^x e^y$.
23. Show that $e^{ix} = \cos x + i \sin x$.
24. Show that $\sin^2 x + \cos^2 x = 1$.
25. Show that $\sin(x+y) = \sin x \cos y + \cos x \sin y$.
26. Show that $\cos(x+y) = \cos x \cos y - \sin x \sin y$.
27. Show that $e^{\ln x} = x$.
28. Show that $\ln(e^x) = x$.

2 Vocabulary

Define the following terms.

1. relation
2. symmetric (relation)
3. reflexive (relation)
4. transitive (relation)
5. equivalence (relation)
6. cover
7. partition

8. equivalence class
9. operation
10. commutative
11. associative
12. identity
13. inverse
14. monoid without identity
15. monoid
16. group
17. ring without identity
18. ring
19. division ring
20. commutative ring
21. field
22. integral domain
23. field of fractions
24. homomorphism (of monoids)
25. homomorphism (of rings)
26. polynomial ring
27. ring of formal power series
28. ring of Laurent polynomials
29. $\mathbb{F}(x)$
30. $\mathbb{F}((x))$
31. evaluation homomorphism
32. e^x
33. $\sin x$
34. $\cos x$
35. $\ln x$