MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 1
DUE September 13, 2004

Problem A. Numbers

(1) What are the positive integers and why do we care?

(2) What are the nonnegative integers and why do we care?

(3) What are the rational numbers and why do we care?

(4) What are the real numbers and why do we care?

(5) What are the complex numbers and why do we care?

Problem B. Computing with complex numbers

(1) Find a complex number z such that $z + w = w$ for all other complex numbers w.

(2) Find a complex number x such that $xw = w$ for all other complex numbers w.

(3) Compute $(3 - 7i) + (2 + 5i)$ and graph the result.

(4) Compute $(-12 + 3i) - (7 - 5i)$ and graph the result.

(5) Compute $(4 + 8i)(2 - 3i)$ and graph the result.

(6) Compute $\dfrac{-15 + i}{4 + 2i}$ and graph the result.

(7) Compute $(3 - 2i)^3$ and graph the result.

(8) Compute $\sqrt{2i}$ and graph the result.

(9) Compute $\dfrac{1}{a + bi}$ and graph the result, where $a, b \in \mathbb{R}$.

(10) Compute $(3 - 5i) + (7 + 2i)$ and graph the result.

(11) Compute $(5 - 2i) - (3 - 6i)$ and graph the result.

(12) Compute $(2 - 4i)(3 + 2i)$ and graph the result.

(13) Compute $\dfrac{6 - i}{4 + 2i}$ and graph the result.

Problem C. Vocabulary

(1) What is $\sin x$?

(2) What is $\cos x$?

(3) What is $\tan x$?

(4) What is $\cot x$?

(5) What is $\sec x$?

(6) What is $\csc x$?

(7) What is π and where did it come from?

(8) Explain how to measure angles in radians, in degrees, and how convert from degrees to radians.

(9) What is the connection between measuring angles in radians and measuring distances?

(10) What is the circumference of a circle of radius r? How do you kn ow?

(11) What is the length of an arc of angle θ on the boundary of a circle of radius r? How do you know?

(12) What is the area of a circle of radius r? How do you know?

(13) What is the area of a sector of angle θ in a circle of radiu s r? How do you know?

(14) Show that $\sin(-x) = -\sin x$.

(15) Show that $\cos(-x) = \cos x$.

(16) Show that $\sin^2 x + \cos^2 x = 1$.

Problem D. Computing trigonometric functions

(1) Explain how to derive $\sin \frac{\pi}{6}$, $\cos \frac{\pi}{6}$, $\tan \frac{\pi}{6}$, $\cot \frac{\pi}{6}$, $\sec \frac{\pi}{6}$ and $\csc \frac{\pi}{6}$ in radical form.

(2) Explain how to derive $\sin\frac{\pi}{3}$, $\cos\frac{\pi}{3}$, $\tan\frac{\pi}{3}$, $\cot\frac{\pi}{3}$, $\sec\frac{\pi}{3}$ and $\csc\frac{\pi}{3}$ in radical form .

(3) Explain how to derive $\sin\frac{\pi}{4}$, $\cos\frac{\pi}{4}$, $\tan\frac{\pi}{4}$, $\cot\frac{\pi}{4}$, $\sec\frac{\pi}{4}$ and $\csc\frac{\pi}{4}$ in radical form .

(4) Explain how to derive $\sin\frac{\pi}{2}$, $\cos\frac{\pi}{2}$, $\tan\frac{\pi}{2}$, $\cot\frac{\pi}{2}$, $\sec\frac{\pi}{2}$ and $\csc\frac{\pi}{2}$ in radical form .

(5) Explain how to derive $\sin 0$, $\cos 0$, $\tan 0$, $\cot 0$, $\sec 0$ and $\csc 0$ in radical form.

(6) Explain how to derive $\sin\frac{3\pi}{4}$, $\cos\frac{3\pi}{4}$, $\tan\frac{3\pi}{4}$, $\cot\frac{3\pi}{4}$, $\sec\frac{3\pi}{4}$ and $\csc\frac{3\pi}{4}$ in radical f orm.

(7) Explain how to derive $\sin\frac{-2\pi}{3}$, $\cos\frac{-2\pi}{3}$, $\tan\frac{-2\pi}{3}$, $\cot\frac{-2\pi}{3}$, $\sec\frac{-2\pi}{3}$ and $\csc\frac{-2\pi}{3}$ in radica l form.

(8) Compute $\sin\frac{\pi}{6} + \cos\frac{\pi}{6}$ in radical form.

(9) Compute $\sin\frac{\pi}{4} + \cos\frac{\pi}{3}$ in radical form.

(10) Compute $\sin\frac{\pi}{4} - \sin\frac{\pi}{2}$ in radical form.

(11) Compute $\left(\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{6}\right)$ in radical form.

(12) Compute $\left(\tan\frac{\pi}{3}\right)\left(\cot\frac{\pi}{3}\right)$ in radical form.

Problem E. Trigonometric function identities

(1) Verify the identity $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$.

(2) Verify the identity $\sin(x/2) = \pm\sqrt{\dfrac{1-\cos x}{2}}$.

(3) Verify the identity $\cos 3x = \cos^3 x - 3\cos x \sin^2 x$.

(4) Verify the identity $\sin 3x = 3\cos^2 x \sin x - \sin^3 x$.

(5) Verify the identity $\sin^2 A \cot^2 A = (1 - \sin A)(1 + \sin A)$.

(6) Verify the identity $\tan B = \dfrac{\cos B}{\sin B \cot^2 B}$.

(7) Verify the identity $\dfrac{\tan V \cos V}{\sin V} = 1$.

(8) Verify the identity $\sin E \cot E + \cos E \tan E = \sin E + \cos E$.

(9) Verify the identity $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x} - 1 = 0$.

(10) Verify the identity $\dfrac{\sec A - 1}{\sec A + 1} + \dfrac{\cos A - 1}{\cos A + 1} = 0$.

(11) Verify the identity $\sin V(1 + \cot^2 V) = \csc V$.

(12) Verify the identity $\dfrac{\sin(\pi/2 - w)}{\cos(\pi/2 - w)} = \cot w$.

(13) Verify the identity $\sec(\pi/2 - z) = \dfrac{1}{\sin z}$.

(14) Verify the identity $1 + \tan^2(\pi/2 - x) = \dfrac{1}{\cos^2(\pi/2 - x)}$.

(15) Verify the identity $\dfrac{\sin A}{\csc A} + \dfrac{\cos A}{\sec A} = 1$.

(16) Verify the identity $\dfrac{\sec B}{\cos B} - \dfrac{\tan B}{\cot B} = 1$.

(17) Verify the identity $\dfrac{1}{\csc^2 w} + \sec^2 w + \dfrac{1}{\sec^2 w} = 2 + \dfrac{\sec^2 w}{\csc^2 w}$.

(18) Verify the identity $\sec^4 V - \sec^2 V = \dfrac{1}{\cot^4 V} + \dfrac{1}{\cot^2 V}$.

(19) Verify the identity $\sin^4 x + \cos^2 x = \cos^4 x + \sin^2 x$.

(20) Verify the identity $\tan 3\alpha = \dfrac{3\tan \alpha - \tan^3 \alpha}{1 - 3\tan^2 \alpha}$.

(21) Verify the identity $\cot(\alpha/2) = \dfrac{\sin \alpha}{1 - \cos \alpha}$.

(22) Verify the identity $\cos(\pi/6 - x) + \cos(\pi/6 + x) = \sqrt{3}\cos x$.

(23) Verify the identity $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$.

(24) Verify the identity $\sin(\pi/3 - x) + \sin(\pi/3 + x) = \sqrt{3}\cos x$.

(25) Verify the identity $\cos(\pi/4 - x) - \cos(\pi/4 + x) = \sqrt{2}\sin x$.

(26) Verify the identity $2\sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$.

(27) Verify the identity $2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$.

Problem F. Fun trigonometric function identities

(1) Verify the identity $\cos 2\theta = 2\sin(\pi/4 + \theta)\sin(\pi/4 - \theta)$.

(2) Verify the identity $(1/2)\sin 2A = \dfrac{\tan A}{1 + \tan^2 A}$.

(3) Verify the identity $\cot(x/2) = \dfrac{1 + \cos x}{\sin x}$.

(4) Verify the identity $\sin 2B(\cot B + \tan B) = 2$.

(5) Verify the identity $\dfrac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$.

(6) Verify the identity $1 + \cos 2A = \dfrac{2}{1 + \tan^2 A}$.

(7) Verify the identity $\tan 2x \tan x + 2 = \dfrac{\tan 2x}{\tan x}$.

(8) Verify the identity $\csc A \sec A = 2\csc 2A$.

(9) Verify the identity $\cot x = \dfrac{\sin 2x}{1 - \cos 2x}$.

(10) Verify the identity $1 - \sin A = \left(\sin\dfrac{A}{2} - \cos\dfrac{A}{2}\right)^2$.

(11) Verify the identity $\cos^4 A = \dfrac{2\cos 2A + \cos^2 2A + 1}{4}$.

(12) Verify the identity $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \dfrac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$.

(13) Verify the identity $\dfrac{\sin\alpha + \sin 3\alpha}{\cos\alpha + \cos 3\alpha} = \tan 2\alpha$.

(14) Verify the identity $\dfrac{\cos 2A}{1 + \sin 2A} = \dfrac{\cot A - 1}{\cot A + 1}$.

(15) Verify the identity $\dfrac{\cos A + \sin A}{\cos A - \sin A} = \dfrac{1 + \sin 2A}{\cos 2A}$.

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(16) Verify the identity $\cot \alpha - \cot \beta = \dfrac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta}$.

(17) Verify the identity $\tan \theta \csc \theta \cos \theta = 1$.

(18) Verify the identity $\cos^2 \theta = \dfrac{\cot^2 \theta}{1 + \cot^2 \theta}$.

(19) Verify the identity $\dfrac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2$.

(20) Verify the identity $(\tan A - \cot A)^2 + 4 = \sec^2 A + \csc^2 A$.

(21) Verify the identity $\cos B \cos(A + B) + \sin B \sin(A + B) = \cos A$.

(22) Verify the identity $\dfrac{\tan A - \sin A}{\sec A} = \dfrac{\sin^3 A}{1 + \cos A}$.

(23) Verify the identity $\dfrac{2\tan^2 A}{1 + \tan^2 A} = 1 - \cos 2A$.

(24) Verify the identity $\tan 2A = \tan A + \dfrac{\tan A}{\cos 2A}$.

(25) Verify the identity $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$.

(26) Verify the identity $\dfrac{4\sin A}{1 - \sin^2 A} = \dfrac{1 + \sin A}{1 - \sin A} - \dfrac{1 - \sin A}{1 + \sin A}$.

(27) Verify the identity $\tan A + \sin A = \dfrac{\csc A + \cot A}{\csc A \cot A}$.