MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 14
DUE December 13, 2004

Problem A. Derivatives with all functions mixed together.

(1) Find $\frac{dy}{dx}$ when $y = \frac{2\tan x}{\tan x + \cos x}$.

(2) Find $\frac{dy}{dx}$ when $y = \sqrt{x \sin x}$.

(3) Find $\frac{dy}{dx}$ when $y = \frac{x + \sin 2x}{\cos 3x}$.

(4) Find $\frac{dy}{dx}$ when $y = e^{5x} \ln(\sec x)$.

(5) Find $\frac{dy}{dx}$ when $y = \frac{x^5}{\sin^{-1} 2x}$.

(6) Find $\frac{dy}{dx}$ when $y = \sin x^2 - \frac{\tan x}{1 + x^2}$.

(7) Find $\frac{dy}{dx}$ when $y = (\tan \sqrt{x} + x^2 - \sin x)^3$.

(8) Find $\frac{dy}{dx}$ when $y = \frac{\sin^3 x \cos^3 x}{\cos 3x}$.

(9) Find $\frac{dy}{dx}$ when $y = e^x \tan x + \frac{\ln x}{\sin x}$.

(10) Find $\frac{dy}{dx}$ when $y = 2a^x \ln x$.

(11) Find $\frac{dy}{dx}$ when $y = \frac{x^{3/2} + 1}{3\sqrt{x}}$.

(12) Find $\frac{dy}{dx}$ when $y = \frac{1 - x^2}{x^2 - x + 1}$.

(13) Find $\frac{dy}{dx}$ when $y = \sqrt{1 + \ln x \ln \sin x}$.

(14) Find $\dfrac{dy}{dx}$ when $y = 7x^{1/2} + 5x^{-7/2} + \sin^{-1}(x^4) - \ln\cot x$.

(15) Find $\dfrac{dy}{dx}$ when $y = \sin^2 x \cos^3 x$.

(16) Find $\dfrac{dy}{dx}$ when $y = \sin mx \cos nx$.

(17) Find $\dfrac{dy}{dx}$ when $y = \sin^m x \cos^n x$.

(18) Find $\dfrac{dy}{dx}$ when $y = \cos^{-1}(1 - 2x^2)$.

(19) Find $\dfrac{dy}{dx}$ when $y = \sin^{-1}(3x - 4x^3)$.

(20) Find $\dfrac{dy}{dx}$ when $y = \dfrac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$.

(21) Find $\dfrac{dy}{dx}$ when $y = (1+x)(1+2x)(1+3x)(1+4x)$.

(22) Find $\dfrac{dy}{dx}$ when $y = \tan^2\sqrt{1-x^2}$.

(23) Find $\dfrac{dy}{dx}$ when $y = \dfrac{\tan x}{x^2}$.

(24) Find $\dfrac{dy}{dx}$ when $y = \dfrac{e^{2x}}{\ln x}$.

(25) Find $\dfrac{dy}{dx}$ when $y = \dfrac{e^{x^2}\tan^{-1} x}{\sqrt{1+x^2}}$.

(26) Find $\dfrac{dy}{dx}$ when $y = e^{\sqrt{x}+2} - e^{\sqrt{x+2}}$.

(27) Find $\dfrac{dy}{dx}$ when $y = 7^{x^2+2x}$.

(28) Find $\dfrac{dy}{dx}$ when $y = \cot^2(e^{3x}x^x)$.

(29) Find $\dfrac{dy}{dx}$ when $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$.

(30) Find $\dfrac{dy}{dx}$ when $y = \ln(\tan^{-1} x)$.

(31) Find $\dfrac{dy}{dx}$ when $y = \csc^{-1}\left(\dfrac{1+x^2}{2x}\right)$.

(32) Find $\dfrac{dy}{dx}$ when $y = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$.

(33) Find $\dfrac{dy}{dx}$ when $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$.

(34) Find $\dfrac{dy}{dx}$ when $y = \sin^{-1} x + \sin^{-1}\sqrt{1-x^2}$.

(35) Find $\dfrac{dy}{dx}$ when $y = \sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$.

(36) Find $\dfrac{dy}{dx}$ when $y = x\sin^{-1} x + \sqrt{1-x^2}$.

(37) Find $\dfrac{dy}{dx}$ when $y = x\cos^{-1} 2x - \frac{1}{2}\sqrt{1-4x^2}$.

(38) Find $\dfrac{dy}{dx}$ when $y = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\tan(x/2)\right)$.

(39) Find $\dfrac{dy}{dx}$ when $y = \tan^{-1}(\sec x + \tan x)$.

(40) Find $\dfrac{dy}{dx}$ when $y = \dfrac{x\cos^{-1} x}{\sqrt{1-x^2}}$.

(41) Find $\dfrac{dy}{dx}$ when $y = \dfrac{1}{2}x\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}(x/a)$.

(42) Find $\dfrac{dy}{dx}$ when $y = \sin^{-1}(2x\sqrt{1-x^2})$.

(43) Find $\dfrac{dy}{dx}$ when $y = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$.

(44) Find $\dfrac{dy}{dx}$ when $y = x^3\sin 2x + \dfrac{\cos x}{x+1}$.

(45) Find $\dfrac{dy}{dx}$ when $y = x^4\sin 2x + \dfrac{x^2}{x^3+1}$.

(46) Find $\dfrac{dy}{dx}$ when $y = x^{\ln x}$.

(47) Find $\dfrac{dy}{dx}$ when $y = (\tan x)^{\cot x}$.

(48) Find $\dfrac{dy}{dx}$ when $y = (\tan x)^{\cot x} + (\cot x)^{\tan x} + x^{\cos^{-1} x}$.

(49) Find $\dfrac{dy}{dx}$ when $y = (\sin x)(e^x)(\ln x)(x^x)(x^{\cos^{-1} x})$.

(50) Find $\dfrac{dy}{dx}$ when $x^y = y^x$.

(51) Find $\dfrac{dy}{dx}$ when $x^{2/3} + y^{2/3} = a^{2/3}$.

(52) Find $\dfrac{dy}{dx}$ when $e^{xy} - 4xy = 0$.

(53) Find $\dfrac{dy}{dx}$ when $xy = \sin(x + y)$.

(54) Find $\dfrac{dy}{dx}$ when $\dfrac{x^m}{a^m} + \dfrac{y^m}{b^m} = 1$.

(55) Find $\dfrac{dy}{dx}$ when $x^m y^n = (x + y)^{m+n}$.

(56) Find $\dfrac{dy}{dx}$ where $y \ln x = x - y$.

Problem B. Integrals with mixed functions.

(1) $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$

(2) $\displaystyle\int \frac{\sin(2 \tan^{-1} x)}{1 + x^2}\, dx$

(3) $\displaystyle\int \frac{\cos(\ln x)}{x}\, dx$

(4) $\displaystyle\int \frac{\csc^2(\ln x)}{x}\, dx$

(5) $\int e^{\tan x} \sec^2 x \; dx$

(6) $\int e^{\cos^2 x} \sin 2x \; dx$

(7) $\int \cot x \ln(\sin x) \; dx$

(8) $\int \dfrac{\cot x}{\ln(\sin x)} \; dx$

(9) $\int \sec x \ln(\sec x + \tan x) \; dx$

(10) $\int \dfrac{x \tan^{-1} x^2}{1 + x^4} \; dx$

(11) $\int \dfrac{x \sin^{-1} x^2}{\sqrt{1 - x^4}} \; dx$

(12) $\int \dfrac{1}{\sqrt{1 - x^2} \sin^{-1} x} \; dx$

(13) $\int \dfrac{1 + \tan x}{x + \ln(\sec x)} \; dx$

(14) $\int \dfrac{\sec x \csc x}{\ln(\tan x)} \; dx$

(15) $\int \dfrac{dx}{x \cos^2(1 + \ln x)}$

(16) $\int e^{-x} \csc^2(2e^{-x} + 5) \; dx$

(17) $\int x^2 e^{x^3} \cos e^{x^3} \; dx$

(18) $\int \dfrac{e^{m \tan^{-1} x}}{1 + x^2} \; dx$

(19) $\int \dfrac{(x + 1)e^x}{\cos^2(xe^x)} \; dx$

(20) $\displaystyle\int \frac{e^{\sqrt{x}} \cos e^{\sqrt{x}}}{\sqrt{x}}\, dx$

Problem C. Areas of regions.

(1) Find the area inside the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

(2) Using integration find the area of the triangle with vertices $(-1, 1)$, $(0, 5)$ and $(3, 2)$.

(3) Graph the region $\{(x, y) \mid 4x^2 + 9y^2 \leq 36\}$ and find its area.

(4) Find the area of the region $\{(x, y) \mid y^2 \leq 8x, x^2 + y^2 \leq 9\}$.

(5) Find the area of the region $\{(x, y) \mid y^2 \leq x, x^2 + y^2 \leq 2\}$.

(6) Find the area of the region $\{(x, y) \mid x^2 + y^2 \leq 2ax, y^2 \geq ax, x \geq 0, y \geq 0\}$.

(7) Find the area of the region $\{(x, y) \mid y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$.

(8) Find the area of the region $\{(x, y) \mid x^2 + y^2 \leq 1 \leq x + y\}$.

(9) Find the area of the region $\{(x, y) \mid 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

(10) Find the area of the region $\{(x, y) \mid x^2 \leq y \leq |x|\}$.

Problem D. Different types of volume problems.

(1) A solid is generated by rotating, about the x-axis, the area bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$, $x = b$. Its volume, for all $b > a$, is $b^2 - ab$. Find $f(x)$.

(2) A solid is generated by rotating the curve $y = f(x)$, $0 \leq x \leq a$, about the x-axis. Its volume, for all a, is $a^2 + a$. Find $f(x)$.

(3) The area bounded by the curve $y^2 = 4x$ and the straight line $y = x$ is rotated about the x-axis. Find the volume generated.

(4) Sketch the area bounded by the curve $y^2 = 4ax$, the line $x = a$, and the x-axis. Find the volumes generated by rotating this area in each of the following ways:
   (a) about the x-axis,
   (b) about the line $x = a$.
   (c) about the y-axis,

(5) The area bounded by the curve $y = x/\sqrt{x^3+8}$, the x-axis, and the line $x = 2$ is rotated about the y-axis. Compute the volume.

(6) Find the volume of the solid produced by rotating the larger area bounded by $y^2 = x - 1$, $x = 3$ and $y = 1$ about the y-axis.

(7) The area bounded by the curve $y^2 = 4ax$ and the line $x = a$ is rotated about the line $x = 2a$. Find the volume generated.

(8) A twisted solid is generated as follows: We are given a fixed line L in space, and a square of side length s in a plane perpendicular to L. One vertex of the square is on L. As this vertex moves a distance h along L, the square turns through a full revolution, with L as the axis. Find the volume generated.

(9) A twisted solid is generated as follows: We are given a fixed line L in space, and a square of side length s in a plane perpendicular to L. One vertex of the square is on L. As this vertex moves a distance h along L, the square turns through *two* full revolutions, with L as the axis. Find the volume generated.

(10) Two circles have a common diameter and lie in perpendicular planes. A square moves so that its plane is perpendicular to this diameter and its diagonals are chords of the circles. Find the volume generated.

(11) Find the volume generated by rotating the area bounded by the x-axis and one arch of the curve $y = \sin 2x$ about the x-axis.

(12) A round hole of radius $\sqrt{3}$ ft is bored through the center of a solid sphere of radius 2 ft. How much volume is cut out?