MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2004

HOMEWORK 6: SELECTED ANSWERS

Problem B. Where is a function continuous?

(1) all x	(2) all x	(3) $x \neq 0$
(4) $x \neq 0$	(5) $k = 2/5$	(6) $1 \leq x \leq 3$
(7) $x \neq 0$	(8) $x \neq 0$	(9) $a = -2$
(10) $x \geq 0, x \neq 1$	(11) all x	(12) $a = 3$
(13) $x \neq a$	(14) $x \neq 0$	(15) all x
(16) all x	(18) all x	(19) x not an integer
(20) $x \neq 1$	(21) $-1 \leq x \leq 2$	

Problem D. Increasing, decreasing, and concavity.

(9) 1 (10) $a = 3$ and $b = 5$

Problem E. Graphing polynomials.

(1) Defined for all x; continuous for all x; differentiable for all x; increasing nowhere; decreasing nowhere; concave up nowhere; concave down nowhere; all x are critical points; no points of inflection; $y = a$ is an asymptote.

(2) Defined for all x; continuous for all x; differentiable for all x; increasing for all x if $a > 0$; decreasing for all x if $a < 0$; concave up nowhere; concave down nowhere; all x are critical points if $a = 0$ and there are no critical points if $a \neq 0$; no points of inflection; $y = ax + b$ is an asymptote.

(3) Same as (2).

(4) Defined for $x \geq 0$; continuous for $x \geq 0$; differentiable for $x \neq 1$; increasing for $0 < x < 1$; decreasing for $x > 1$; concave up nowhere; concave down nowhere; critical points at $x = 1$ and $x = 0$; no points of inflection; $y = 2 - x$ is an asymptote as $x \to \infty$.

(5) Defined for all x; continuous for all x; differentiable for $x \neq 0$; increasing for $x > 0$; decreasing for $x < 0$; concave up nowhere; concave down nowhere; critical point at $x = 0$; no points of inflection; $y = 2 + x$ is an asymptote as $x \to \infty$, $y = 2 - x$ is an asymptote as $x \to -\infty$.

(6) Defined for all x; continuous for all x; differentiable for $x \neq 1$; increasing for $x > 1$; decreasing for $x < 1$; concave up for $x > 1$; concave down nowhere; critical point at $x = 1$; no points of inflection; $y = 1 - x$ is an asymptote as $x \to -\infty$.

(7) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 1$; decreasing for $x > 1$; concave up nowhere; concave down for all x; critical point at $x = 1$; no points of inflection; no asymptotes.

(8) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 1/2$; decreasing for $x > 1/2$; concave up nowhere; concave down for all x; critical point at $x = 1/2$; no points of inflection; no asymptotes.

(9) Defined for all x; continuous for all x; differentiable for all x; increasing for $x > 1/3$; decreasing for $x < 1/3$; concave up for all x; concave down nowhere; -critical point at $x = 1/3$; no points of inflection; no asymptotes.

(10) Defined for all x; continuous for all x; differentiable for all x; increasing for all x; decreasing nowhere; concave up for $x > 0$; concave down for $x < 0$; critical point at $x = 0$; point of inflection at $x = 0$; no asymptotes.

(11) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < -1/\sqrt{3}, x > 1/\sqrt{3}$; decreasing for $-1/\sqrt{3} < x < 1/\sqrt{3}$; concave up for $x > 0$; concave down for $x < 0$; critical points at $x = \pm 1/\sqrt{3}$; point of inflection at $x = 0$; no asymptotes.

(12) Same as (11).

(13) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 4/3, x > 2$; decreasing for $4/3 < x < 2$; concave up for $x > 5/3$; concave down for $x < 5/3$; critical points at $x = 2$ and $x = 4/3$; point of inflection at $x = 5/3$; no asymptotes.

(14) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 1, x > 6$; decreasing for $1 < x < 6$; concave up for $x > 7/2$; concave down for $x < 7/2$; critical points at $x = 6$ and $x = 1$; point of inflection at $x = 7/2$; no asymptotes.

(15) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < -5/3, x > 2$; decreasing for $-5/3 < x < 2$; concave up for $x > 1/6$; concave down for $x < 1/6$; critical points at $x = -5/3$ and $x = 2$; point of inflection at $x = 1/6$; no asymptotes.

(16) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 0$; decreasing for $x > 0$; concave up nowhere; concave down for all x; critical points at $x = 0$; no points of inflection; no asymptotes.

(17) Defined for all x; continuous for all x; differentiable for all x; increasing for $-1 < x < 0$ and $x > 2$; decreasing for $x < -1$ and $0 < x < 2$; concave up for x less than about $-1/2$, and for x greater than about 1.2; concave down for x between about $-1/2$ and 1.2; critical points at $x = -1$, $x = 0$ and $x = 2$; points of inflection at about $-1/2$ and about 1.2; no asymptotes.

(18) Defined for all x; continuous for all x; differentiable for all x; increasing for $0 < x < 1$ and $x > 3$; decreasing for $x < 0$ and $1 < x < 3$; concave up for x less than about $1/2$, and for x greater than about 2.2; concave down for x between about $1/2$ and 2.2; critical points at $x = 0$, $x = 1$ and $x = 3$; points of inflection at about $1/2$ and about 2.2; no asymptotes.

(20) Defined for all x; continuous for all x; differentiable for all x; increasing for $x < 1.2$ and $x > 2$; decreasing for $1.2 < x < 2$; concave up for x between 0 and about 1, and for x greater than about 1.3; concave down for $x < 0$ and x between about 1 and 1.3; critical points at $x = 0$, $x = 1.2$ and $x = 2$; points of inflection at $x = 0$ and x about 1 and about 1.3; no asymptotes.

(21) Defined for all x; continuous for all x; differentiable for all x; increasing for $x > 0$ and less than about 1.2 and for $x > 2$; decreasing for $x < 0$ and between about 1.2 and 2; concave up for x less than about $-.9$ and between about $-.5$ and .5 and for x greater than about 1.5; concave down for x between about $-.9$ and $-.5$ and between about .5 and 1.5; critical points at $x = 0$, $x = 2$ and approximately $-.9$ and 1.2; points of inflection at approximately $-.9$, $-.5$, .5 and 1.5; no asymptotes.