05.08.2025
ART Seminar
A. Ram

①

Proof Machine

If ___A___ then ___B___

Assume the ifs
to show the thens.

Formulated to provide a
mechanical way to complete proofs
and organise. I've never been stuck
since, no talent needed.

MAIN IDEA! Distilled grammar.

Proof type I: To show: LHS = RHS

LHS = ... = ... = ... = ... = STUFF

RHS = ... = ... = ... = ... = SAME STUFF.

So LHS = RHS.

<u>Proof type II</u>: To show: If __A__ then __B__ A. Ram

Assume __A__

To show: __B__

<u>Proof type III</u> To show: There exists __E__
such that __F__

Let E = __?__

To show: __F__

Usually completing the proof of __F__
tells you what __?__ is.

This requires a strict grammar:

__F__ must be (a) If __ then __
(b) If __ then __

otherwise proof machine crashes.

for ~~each~~, for ~~every~~, for ~~all~~, for ~~some~~

if, there exists

<u>MAXIM</u>: Don't think, JUST DO IT,
copy the <u>definition</u>.

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Grammar of definitions

Nouns:

A <u>thing</u> T is a ____ such that

(a) If ____ then ____

(b) If ____ then ____

Adjectives: (two forms)

A <u>adjective thing</u> is a thing T such that

(a) If ____ then ____

(b) If ____ then ____

A thing T is <u>adjective</u> if T satisfies:

(a) If ____ then ____

(b) If ____ then ____.

Examples

(1) On my website

(2) We can do some together.

(3) If you don't understand a proof in a book or paper, rewrite it in proof machine (and thus find the structure and the gaps in the reference).

Warnings:

(1) Length

(2) Proof machine is powerful because it is mechanical (menial)

This means proof machine is often not sensitive to audience.

(3) "I feel like I'm reading someone elses code".

(4) It can be emotionally difficult to "accept" a definition.

Favorite proofs to illustrate

(1) If $xy = yx$ then $e^x e^y = e^{x+y}$.

(2) If $f: X \to Y$ is a function then f has an inverse function if and only if f is bijective

(3) If $x, y \in \mathbb{R}_{>0}$ then $x < y$ if and only if $x^2 < y^2$.

(4) The area of a circle is πr^2.

(5) The quadratic formula.

(6) A partition of a set S is equivalent to an equivalence relation on S.

(7) A homomorphism f is injective if and only if $\ker(f) = 0$.

(8) All cosets of H in G are the same size.

(9) Every function f is a composition $f = g \circ h$ where h is injective and g is surjective

(10) Matrix multiplication is associative.

(11) The determinant is a homomorphism.