04.03.2025
ART Seminar
A. Ram

Degenerating

(work in progress w. L. Colmenarejo, L. Gagnon, N. Thiem

DAHA $\rightsquigarrow$ RCA

$K({}^G\mathcal{L})$ $\rightsquigarrow$ $H({}^G\mathcal{L})$

Macdonald polynomials $\rightsquigarrow$??

Philosophy

$$\mathbb{C}[y_1^{\pm 1}, \ldots, y_n^{\pm 1}] = K_T(pt) \xrightarrow{ch} H_T(pt) = \mathbb{C}[y_1, \ldots, y_n]$$

$$y^\lambda = [\mathcal{L}_\lambda] \longmapsto 1 + y_\lambda + \frac{1}{2} y_\lambda^2 + \frac{1}{3!} y_\lambda^3 + \cdots$$

$$y^\lambda \overset{\log}{\rightsquigarrow} y_\lambda = c_1(\mathcal{L}_\lambda)$$

$$\overset{lap}{\rightsquigarrow} 1 + y_\lambda$$

If $\lambda = (\lambda_1, \ldots, \lambda_n)$ then

$y^\lambda = y_1^{\lambda_1} \cdots y_n^{\lambda_n}$ and $y_\lambda = \lambda_1 y_1 + \cdots + \lambda_n y_n$

Degenerations of ECA

ECA

$\swarrow$ $\searrow$

YDTCA DYTCA

$\searrow$ $\swarrow$

RCA

$K \otimes K$

$ch \otimes 1$ $\swarrow$ $\searrow$ $1 \otimes ch$

$H \otimes K$ $K \otimes H$

$1 \otimes ch$ $\searrow$ $\swarrow$ $ch \otimes 1$

$H \otimes H$

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What is the ECA? An algebra H

Generators: T_{s_1}, g, X^{ω}, $q^{\frac{1}{2}}$, $t^{\frac{1}{2}}$

Relations (0) $q^{\frac{1}{2}}, t^{\frac{1}{2}} \in Z(H)$

(1) $T_{s_1}^2 = (t^{\frac{1}{2}} - t^{-\frac{1}{2}})T_{s_1} + 1$, $\quad g^2 = 1$

(2) $T_{s_1} X^{\omega} = X^{-\omega} T_{s_1} - (t^{\frac{1}{2}} - t^{-\frac{1}{2}}) X^{-\omega}$

$g X^{\omega} = q^{\frac{1}{2}} X^{-\omega} g$

The polynomial representation

H acts on $\mathbb{C}[X^{\omega}, X^{-\omega}]$ determined by

$T_{s_1} \cdot 1 = t^{\frac{1}{2}}$, $\quad g \cdot 1 = 1$, $\quad X^{k\omega} \cdot 1 = X^{k\omega}$

for $k \in \mathbb{Z}$. Let $Y^{\omega^\vee} = g T_{s_1}$ and

$E_0 = 1$,

$E_{2\omega} = X^{2\omega} + \frac{(1-t)q}{1-qt}$

$E_{-2\omega} = X^{-2\omega} + \frac{(1-t)}{(1-qt)} + \frac{(1-t)(1-t)q}{(1-q^2t)(1-qt)} + \frac{(1-t)}{(1-q^2t)} X^{2\omega}$

Then

$Y^{\omega^\vee} E_0 = t^{\frac{1}{2}} E_0$, $\quad Y^{\omega^\vee} E_{2\omega} = q^{-1} t^{\frac{1}{2}} E_{2\omega}$, $\quad Y^{\omega^\vee} E_{-2\omega} = q t^{-\frac{1}{2}} E_{-2\omega}$.

<u>What is the YD TCA?</u> An algebra H

<u>Generators</u>: $t_{s_1^\vee}$, $g^\vee$, $y_{\omega^\vee}$, α, c

<u>Relations</u>: ⓪ $\alpha, c \in Z(H)$

① $t_{s_1^\vee}^2 = 1$, $(g^\vee)^2 = 1$

② $t_{s_1^\vee} y_{\omega^\vee} = -y_{\omega^\vee} t_{s_1} + c$

$g^\vee y_{\omega^\vee} = (-y_{\omega^\vee} + \frac{1}{2}\alpha) g^\vee$

<u>The polynomial representation</u>

Let $X^{\omega} = g^\vee t_{s_1}$

Then H acts on $\mathbb{C}[X^{\omega}, X^{-\omega}]$ determined by

$t_{s_1} \cdot 1 = 1$, $y_{\omega^\vee} \cdot 1 = \frac{1}{2}c$, $X^{k\omega} \cdot 1 = X^{k\omega}$

Let $E_0^{(\alpha)} = 1$

$E_\mu^{(\alpha)} = \lim_{t \to 1} E_\mu(t^\alpha, t)$

$E_{2\omega}^{(\alpha)} = X^{2\omega} + \frac{c}{\alpha + c}$

$E_{-2\omega}^{(\alpha)} = X^{-2\omega} + \frac{c}{\alpha + c} + \frac{c^2}{(2\alpha + c)(\alpha + c)} + \frac{c}{2\alpha + c} X^{2\omega}$

Then

$y_{\omega^\vee} E_0^{(\alpha)} = \frac{1}{2}c E_0^{(\alpha)}$, $y_{\omega^\vee} E_{2\omega}^{(\alpha)} = (-\alpha + \frac{1}{2}c) E_{2\omega}^{(\alpha)}$ and

$y_{\omega^\vee} E_{-2\omega}^{(\alpha)} = (\alpha - \frac{1}{2}c) E_{-2\omega}^{(\alpha)}$

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The new generators for the ECA

$$y^{\omega^\vee} = y T_{s_1}, \quad T_{s_1 v} = T_{s_1}, \quad g^\vee = X^{\omega} T_{s_1}, \quad q^{\frac{1}{2}}, t^{\frac{1}{2}}$$

and new relations

(0) $q^{\frac{1}{2}}, t^{\frac{1}{2}} \in Z(H)$

(1) $T_{s_i}^2 = (t^{\frac{1}{2}} - t^{-\frac{1}{2}}) T_{s_i} + 1, \quad (g^\vee)^2 = 1$

(2) $T_{s_1} y^{\omega^\vee} = y^{\omega^\vee} T_{s_1 v} + (t^{\frac{1}{2}} - t^{-\frac{1}{2}}) y^{\omega^\vee}$

$g^\vee y^{\omega^\vee} = q^{-\frac{1}{2}} y^{-\omega^\vee} g^\vee$

The conversion: ECA to YO TCA

$$y^{\omega^\vee} = e^{r y_{\omega^\vee}}, \quad q^{\frac{1}{2}} = e^{\frac{1}{2} r d}, \quad t^{\frac{1}{2}} = e^{\frac{1}{2} r c}$$

$$T_{s_1 v} + \frac{t^{-\frac{1}{2}}(1-t)}{1 - y^{2\omega^\vee}} = t_{s_1 v} + \frac{c}{y_{-2\omega^\vee}}, \qquad g^\vee = g^\vee.$$

The module $L_{3/2}$: Let $t = q^{-3/2}$

The polynomial rep. has a submodule

$$N = HE_{4\omega} \text{ where } E_{4\omega} = \dots$$

Then

$$L_{3/2} = \frac{\mathbb{C}[X^{2\omega}, X^{-2\omega}]}{N} \text{ has basis } \{E_0, E_{2\omega}, E_{-2\omega}\}$$

and, with respect to this basis

$$y^{\omega^\vee} = \begin{pmatrix} t^{\frac{1}{2}} & 0 & 0 \\ 0 & q^{-1}t^{-\frac{1}{2}} & 0 \\ 0 & 0 & qt^{\frac{1}{2}} \end{pmatrix} \quad \tau_1^\vee = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{(1-q^{-2})(1-q^2t^2)}{(1-q^{-2}t^{-1})(1-q^2t)} \\ 0 & 1 & 0 \end{pmatrix}$$

where $\tau_1^\vee = T_{s_1^\vee} + \dfrac{t^{\frac{1}{2}}(1-t)}{1-y^{-2\omega^\vee}}$.

The module $L_{3/2}^{(\alpha)}$: Let $c = -\frac{3}{2}\alpha$

The polynomial rep. has a submodule

$$N^{(\alpha)} = HE_{4\omega}^{(\alpha)} \text{ where } E_{4\omega}^{(\alpha)} = \dots$$

Then

$$L_{3/2}^{(\alpha)} = \frac{\mathbb{C}[x^{2\omega}, x^{-2\omega}]}{N^{(\alpha)}} \text{ has basis } \{E_0^{(\alpha)}, E_{2\omega}^{(\alpha)}, E_{-2\omega}^{(\alpha)}\}$$

and, with respect to this basis

$$y_{\omega^\vee} = \begin{pmatrix} \frac{1}{2}c & 0 & 0 \\ 0 & -\alpha+\frac{1}{2}c & 0 \\ 0 & 0 & \alpha-\frac{1}{2}c \end{pmatrix} \quad \tau_1^\vee = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{2\alpha(2\alpha+2c)}{(2\alpha-c)(2\alpha+c)} \\ 0 & 1 & 0 \end{pmatrix}$$