D-N. VERMA (1933-2012): A MEMORY

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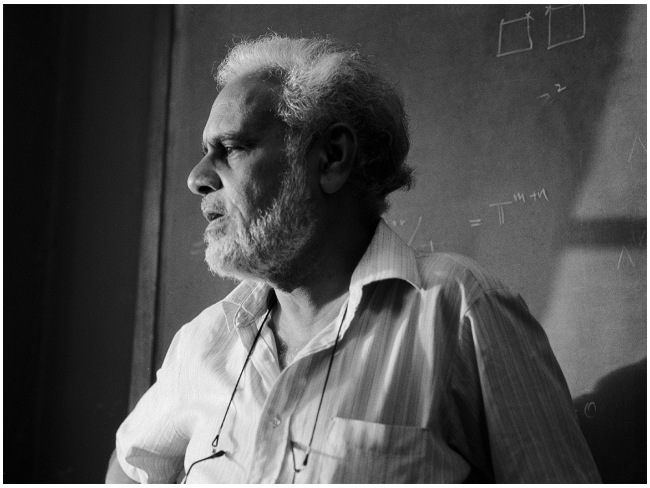

25-6-1933 to 10-6-2012[1]

Who was D.-N. Verma? A character certainly – even more so than most mathematicians. For those of us who dabble in Representation Theory he certainly has had a great effect on us for we cannot imagine a world without Verma modules and the rich theory and structures that they support. It is a great boon for me to know "Verma modules" and also to have known the man D.-N. Verma and the shimmering sea in his mind around these objects.

I can give only a very personal account. To do so I must go back to the year 1966.

D.-N. Verma completed his PhD in 1966 from Yale University under the guidance of N. Jacobson. The title of his thesis was "Structures

[1]Photo credit: R.V. Moody; from his collection at *labyrinth.zenfolio.com*

of Certain Induced Representations of Complex Semisimple Lie Algebras". These induced representations are what are now commonly called "Verma modules". According to my incomplete knowledge (the little I know comes, I think, from discussions long ago with Kostant), these modules had appeared previously in work of Harish-Chandra and Chevalley, but they solidly entered the community psyche after Verma's thesis came to the attention of Kostant and Dixmier. In Russia, at about the same time, the Gelfand school (particularly J. Bernstein, I.M. Gelfand and S. Gelfand) were intensely studying these modules and their work had a great influence in shaping the resulting theory.

I was born in October 1966. I like to say that in 1966-1967, R. Moody, L. Solomon, D.-N. Verma and I were all in Las Cruces, New Mexico USA working hard on research in Lie Theory. It is true that we were all in Las Cruces, New Mexico USA that year, but it is unlikely, given my age, that I was helping very effectively with the research. D.-N. Verma was a good friend of my father and so we certainly met that year. The team of R. Moody, L. Solomon and D.-N. Verma was probably the most promising trio of young Lie theorists of the time. By the time I properly entered Representation Theory in 1988 they had all become legends in the field.

D-N. Verma spent the year 1967-1968 at the Institute of Advanced Study in Princeton and, in 1968 joined the Tata Insitute of Fundamental Research (TIFR) in Mumbai, where he remained, except for a few short periods as visiting Professor in Europe and the USA, until his retirement in 1993-1994. I remember that, as a child, every time that we were in India, we would, of course, spend some weeks in Mumbai to visit my Aunt who lived in Fort, near VT (now CST Station). An important part of our visit was our, usually daily, treks to the TIFR (we would walk across the maidan to catch the TIFR bus) where my father would visit his friends and I would, very happily, play on the beautifully manicured grass, and on the rocks along the ocean. A constant aspect of these outings was the company of Verma Uncle, who was, for me as a child, another one of those pleasant features of our visits to India and the TIFR.

In 1987 I entered graduate school at University of California, San Diego and was, not long after, taking a course in Representation Theory from N. Wallach. I remember talking casually on the phone with my father, who was asking me which courses I was doing. When I told him I was taking Representation Theory he asked me, "Did you learn

Verma modules?" ... it clicked, at that moment I realised who Verma was – indeed "Verma Uncle" was the Verma of Verma modules.

In 1991 I completed my PhD at San Diego, a new, fresh, uncertain, Representation Theorist. That summer my father and I visited family in India, including our usual trip to Mumbai. It was a wonderful and inspiring visit for me. Vermaji took me under his wing for a few weeks and, while my father was off talking physics with his colleagues, Vermaji began to teach me: his picture, answers to my questions, many beautiful vistas of the field that I hadn't imagined.

It must have been wonderful also for Vermaji that summer, as he had too few disciples that could process the flood of haphazard observations and relationships between structures. As I matured, it was also difficult for me in later conversations, as we students become rigid as we get older and don't listen so well. But at that moment, it was ideal, and there was no faster way for me to learn the depths and intricacies of the structures behind BGG resolutions, Jacobi-Trudi formulas and special values of Kazhdan-Lusztig polynomials. And learn I did, fast, and it has had a great influence on all my future work.

From that time D.-N. Verma and I had two relationships: a familial one, as I was the son of his close friend, and a mathematical one. I have had consistent sporadic mathematical contact with him since 1991. As many of his friends know, one will, at periodic intervals, receive a long email and a rambling preprint with many observations and partial theorems and not quite finished connections between important structures. The most recent of these arrived in my Inbox on 14 March of this year.

Looking back at this email I am reminded of discussions with I.M. Gelfand in the late 1990s, which sometimes seemed to me to require infinite patience as he went on rambling about something that I couldn't focus on very well. However, on those few occasions when my willpower was great enough to force myself to focus for the complete story, I was always amazed afterwards at what a treasure of a piece of knowledge I had been given – insights far beyond those occurring in ordinary months of work and learning. Verma was similar. If you had the patience and ability to wade through and parse it, you could be certain there would be a gem there. I remember sitting with Verma on the bus during an excursion on the free afternoon at a conference in Magdeburg in 1998 when he explained to me how the Pittie-Steinberg-Hulsurkar basis for K-theory of flag varieties was the same as the Shi arrangement. This

is another example that has powerfully shaped my view of mathematics (the picture of the Shi arrangement exactly as Verma told it to me appears in my paper in the volume in honour of Steinberg's 80th birthday).

Verma's final email to me was stimulated by the recent passing of our mutual good friend Harsh Pittie. This had motivated him to think again about the Pittie-Steinberg-Hulsurkar picture and the Shi arrangement and its relationships to various structures. His extensive email has many paragraphs on this. Of course he is right, this is fundamentally connected to the Kazhdan-Lusztig theory of affine Weyl groups, cohomology and quantum cohomology of flag varieties, the moduli of stable vector bundles, conformal blocks, The Chevalley-Shephard-Todd theorem, the Knizhnik-Zamolodchikov connection, the moduli space of Riemann surfaces with marked points, the Verlinde formulas, and, in his words, the "Magical Expansion Formulae" (by which he means the formulas (7.1), (5.5), (2.3) and the formula in footnote 2 of his paper, "The role of affine Weyl groups in the representation theory of algebraic Chevalley group and their Lie algebras", in the Proceedings of the 1971 Summer School at Budapest edited by IM Gelfand).

However, I heartily admit that neither he nor I were ever capable of shaping all these connections into a coherent mathematical framework for easy processing by the community. In his email, Verma suggests looking at his Budapest paper (certainly my favourite paper of his for its myriad of realisations). I concur with his suggestion, particularly after having spent a couple of afternoons this past week rereading bits and pieces of this paper. Many stimulating intricacies of the beautiful patterns of crystallographic symmetry and the gems around mathematics that are controlled by it are to be found here – for anyone willing to don their mask and snorkel and swim in the shimmering sea.