The Chevalley-Shephard-Todd theorem

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Abstract

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1 Definition of a reflection group

Let $\mathfrak{h}^*$ be a vector space over a field $\mathbb{F}$ and let $n = \dim(\mathfrak{h}^*)$. A *reflection* is an element $s_\alpha \in GL(\mathfrak{h}^*)$ such that

$$\dim\big((\mathfrak{h}^*)^{s_\alpha}\big) = n-1, \qquad \text{where} \quad (\mathfrak{h}^*)^{s_\alpha} = \{x \in \mathfrak{h}^* \mid s_\alpha x = x\}.$$

A *reflection group* is a finite subgroup W of $GL(\mathfrak{h}^*)$ generated by reflections.

Theorem 1.1. *Let $\mathfrak{h}^*$ be a vector space and let W be a finite subgroup of $GL(\mathfrak{h}^*)$. The following are equivalent*

(a) W is a reflection group, $\quad W = \langle s_\alpha \mid s_\alpha \in W \text{ is a reflection}\rangle$.

(b) $S(\mathfrak{h}^*)^W$ is a polynomial ring, $\quad S(\mathfrak{h}^*)^W = \mathbb{C}[f_1, f_2, \ldots, f_n]$.

(c) $S(\mathfrak{h}^*)$ is a free $S(\mathfrak{h}^*)^W$-module.

References

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