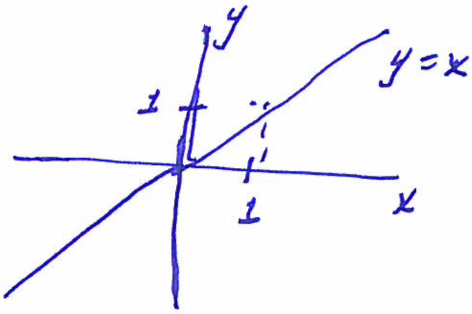
Graphing 10 March 2010

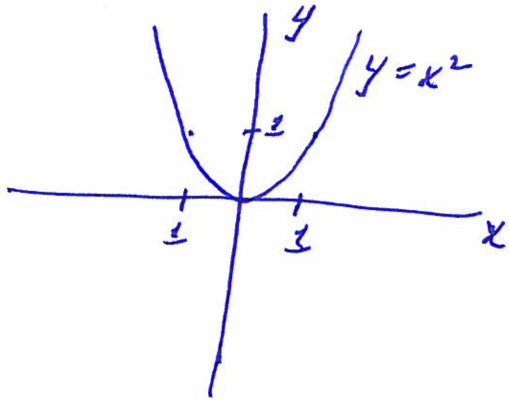
Lecture 5 620-295 Real Analysis with applications

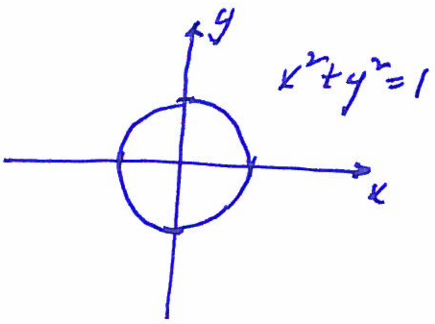
Graphing Techniques

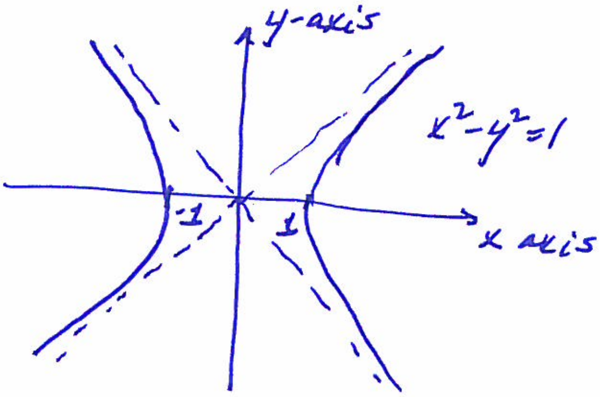
(a) Basic graphs
(b) Shifting
(c) Scaling
(d) Flipping
(e) limits
(f) asymptotes
(g) slopes: Increasing/Decreasing
(h) Concave up/Concave down
points of inflection.

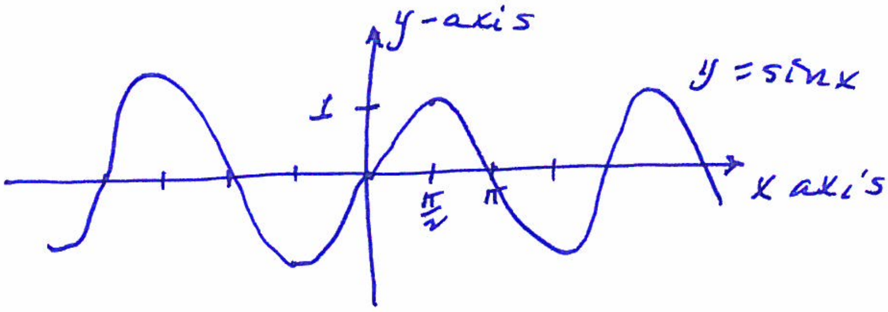
Basic Graphs

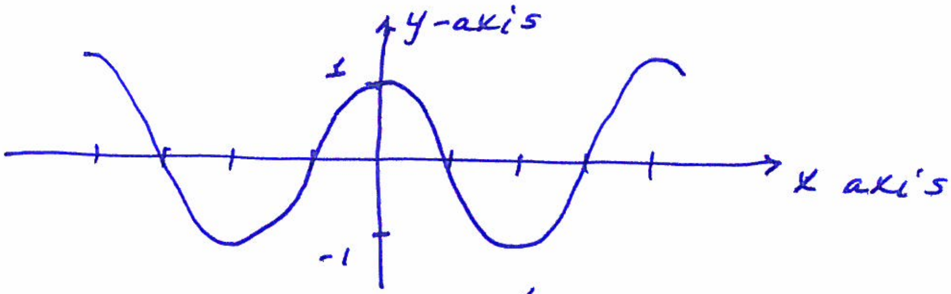


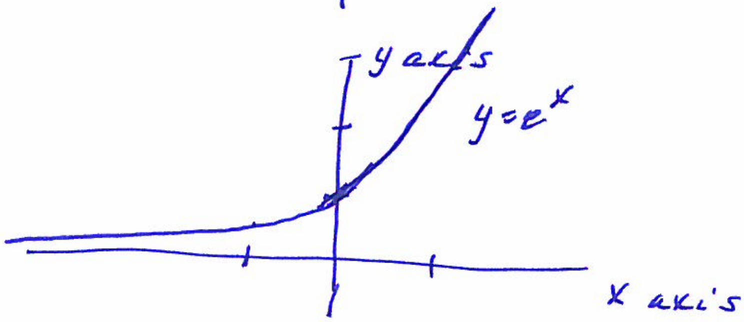




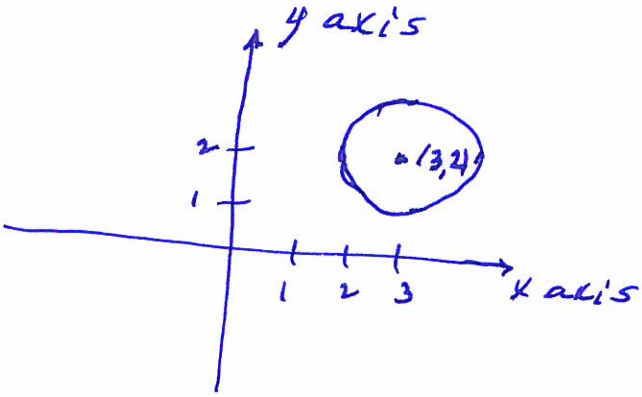








Shifting Graph $(x-3)^2+(y-2)^2=1$.

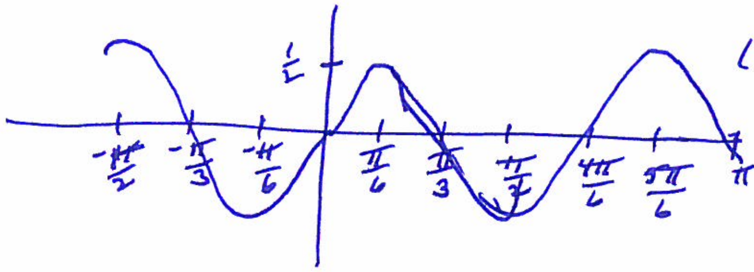


Notes:

(a) $x^2+y^2=1$ is the basic graph, a circle of radius 1

(b) Center is shifted by
3 to the right in the x-direction
2 upwards in the y-direction.

Scaling Graph $2y = \sin 3x$.



Notes:

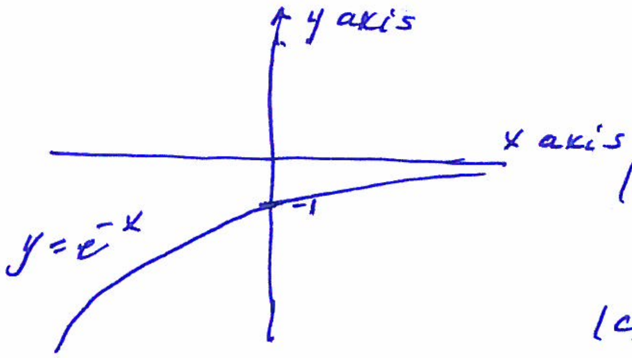
(a) $y = \sin x$ is the basic graph

(b) The x axis is scaled (squished) by 3

(c) The y-axis is scaled (squished) by 2.

Flipping

Example Graph $y = -e^{-x}$.



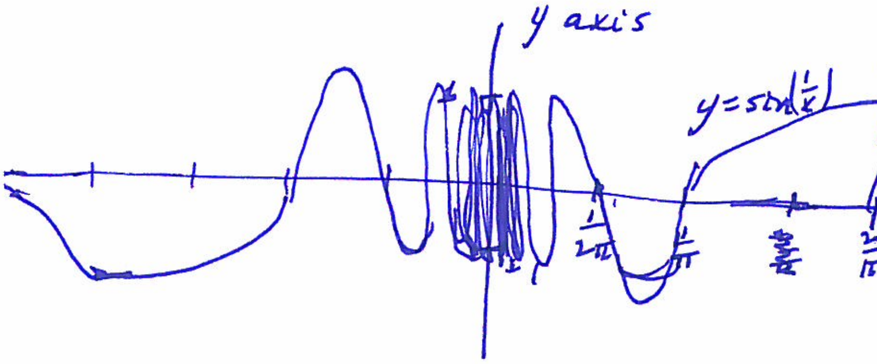
Notes

(a) $y = e^x$ is the basic graph

(b) $y = -e^{-x}$ is the same as $-y = e^{-x}$.

(c) The x-axis is flipped
The y-axis is flipped.

Example Graph $y = \sin(\frac{1}{x})$



Notes

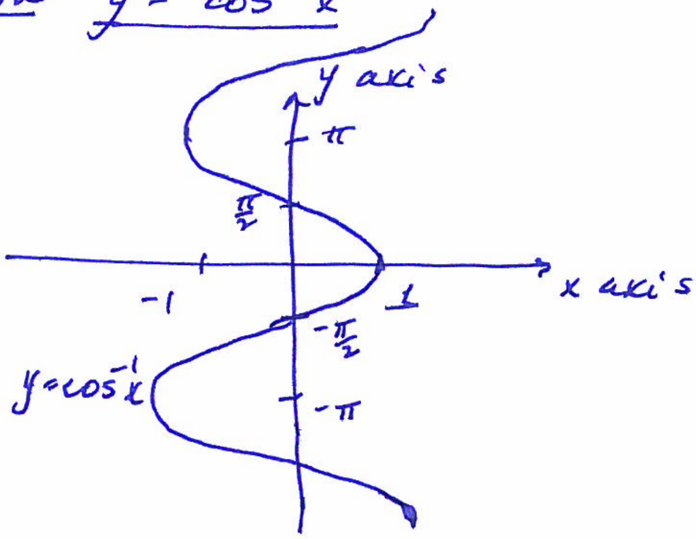
(a) $y = \sin x$ is the basic graph

(b) Positive x axis is flipped in $x = 1$
Negative x-axis is flipped in $x = 1$.

(c) As $x \to \infty$, $\sin(\frac{1}{x}) \to 0^+$
As $x \to -\infty$, $\sin(\frac{1}{x}) \to 0^-$

(d) As $x \to 0^+$, $\sin(\frac{1}{x})$ goes between $+1$ and -1.

Graph $y = \cos^{-1} x$

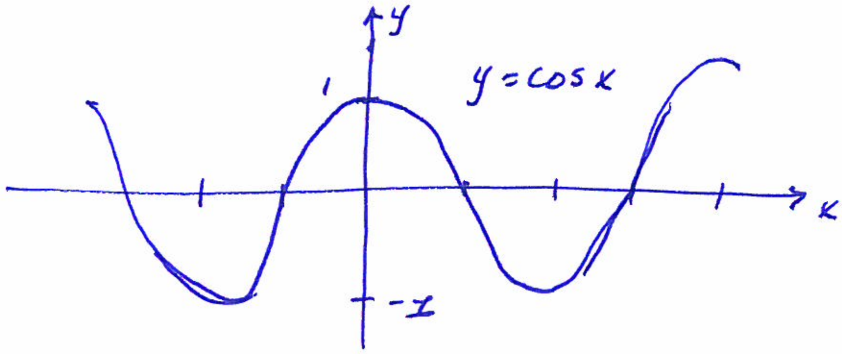


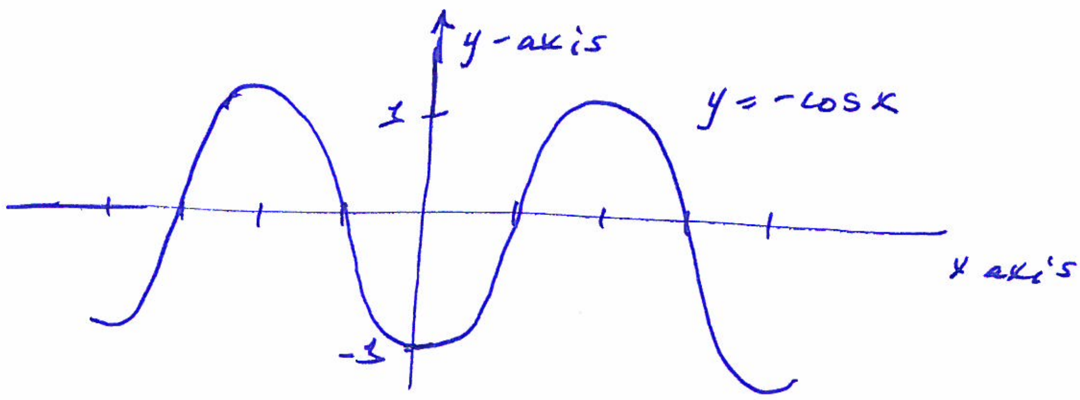
Notes:

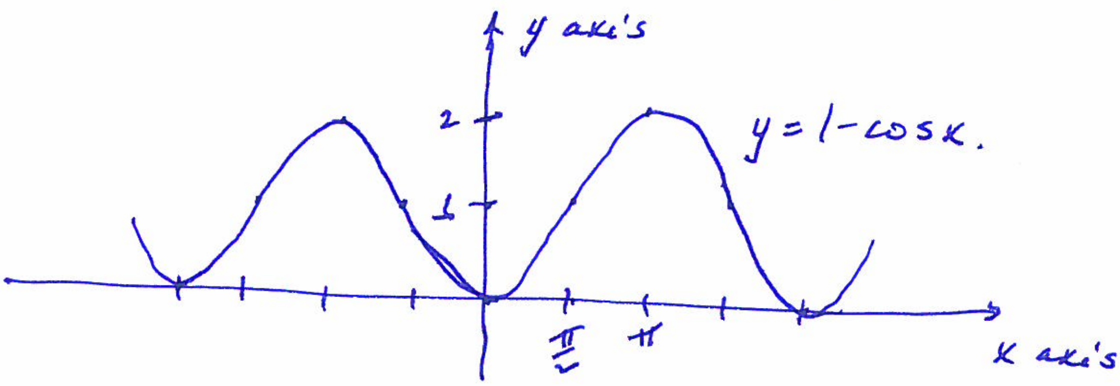
(a) $y = \cos x$ is basic graph

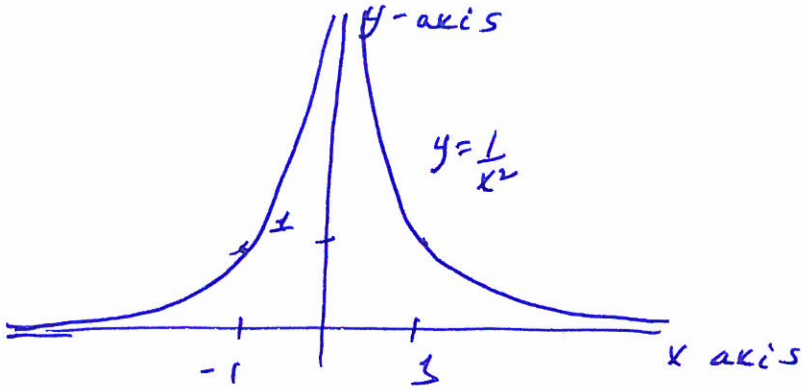
(b) $y = \cos^{-1} x$ is $\cos y = x$ so x and y are switched from the $y = \sin x$ graph.

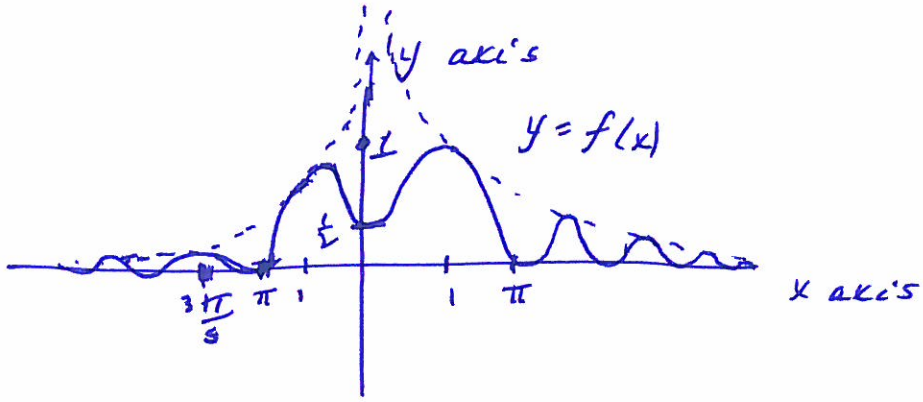
Example
$$f(x) = \begin{cases} \dfrac{1-\cos x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$$











Notes (a) as $x \to 0$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{1-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{\frac{x^2}{2!}-\frac{x^4}{4!}+\frac{x^6}{6!}-\cdots}{x^2} = \lim_{x\to 0} \frac{1}{2!}-\frac{x^2}{4!}+\frac{x^4}{6!}-\cdots = \frac{1}{2}.$$

(if $x = .001$ then $x^2 = .000001$ and $x^4 = .000000000001$)

(b) At $x = 0$, $f(x) = 1$.

(c) At the peaks of $1-\cos x$, $\frac{1-\cos x}{x^2} = \frac{2}{x^2}$

A function is <u>continuous</u> at $x = a$ if it doesn't jump at $x = a$.

A function is <u>continuous</u> at $x = a$ if

$$\lim_{x\to a} f(x) = f(a)$$