Lecture 1 620-295 Real Analysis. 01 March 2010

(1) Location

(2) Announcements

(3) Assessment – The plan is to take exam questions directly from the problem sheets. Assignment 1 due 15 March.

(4) Consultation hours: 2-4:30 in Old Geology 1.

(5) Labs run in weeks 2, 3, 4, and 6.

(6) Text: I highly recommend the book of W. Chen.

Language of Mathematics

Assume

If A then B.

The foundation is definitions.

The explanation is proofs.

It is IMPOSSIBLE to do a proof without knowing the definitions.

Example Prove that $e^x e^y = e^{x+y}$.

The definition of e^x is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Where $k! = k(k-1)(k-2)\cdots 2\cdot 1$, if $k \in \mathbb{Z}_{>0}$.

The definition of $\mathbb{Z}_{>0}$ is

$$\mathbb{Z}_{>0} = \{1, 1+1, 1+1+1, 1+1+1+1, \ldots\}$$

which is often written

$$\mathbb{Z}_{>0} = \{1, 2, 3, 4, \ldots\}.$$

Example inside the example:

$1! = 1$,
$2! = 2$,
$3! = 6$,
$4! = 24$,
$5! = 120$,
$6! = 720$,
$7! = 5040$

So

$$e^1 = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$$

Welcome to Wolfram alpha.

$$e^1 \approx 2.718281828459045235360287471 3$$

(3)

Prove that $e^x e^y = e^{x+y}$.

Righthand side:

$$e^{x+y} = 1 + (x+y) + \frac{(x+y)^2}{2!} + \frac{(x+y)^3}{3!} + \frac{(x+y)^4}{4!} + \cdots$$

$$= 1$$
$$+ x + y$$
$$+ \frac{1}{2}(x^2 + 2xy + y^2)$$
$$+ \frac{1}{6}(x^3 + 3x^2y + 3xy^2 + y^3)$$
$$+ \frac{1}{4!}(x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4)$$
$$+ \frac{1}{5!}(x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5)$$
$$+ \cdots$$

$$= 1$$
$$+ x + y$$
$$+ \frac{1}{2!}x^2 + xy + \frac{1}{2!}y^2$$
$$+ \frac{1}{3!}x^3 + \frac{1}{2!}x^2y + \frac{1}{2!}xy^2 + \frac{1}{3!}y^3$$
$$+ \frac{1}{4!}x^4 + \frac{1}{3!}x^3y + \frac{1}{2!}\frac{1}{2!}x^2y^2 + \frac{1}{3!}xy^3 + \frac{1}{4!}y^4$$
$$+ \frac{1}{5!}x^5 + \frac{1}{4!}x^4y + \frac{1}{2!}\frac{1}{3!}x^3y^2 + \frac{1}{3!}\frac{1}{2!}x^2y^3 + \frac{1}{4!}xy^4 + \frac{1}{5!}y^5$$
$$+ \cdots$$

$$= e^x + e^x y + e^x \frac{1}{2!}y^2 + e^x \frac{1}{3!}y^3 + e^x \frac{1}{4!}y^4 + \cdots$$

Covered under Item 4.7

(4)

$$= e^x\left(1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+\frac{y^5}{5!}+\dots\right)$$

$$= e^x e^y. \quad //.$$

Some more definitions:

$$\sin x = \frac{e^{ix}-e^{-ix}}{2i} \quad \text{with } i^2=-1,$$

$$\cos x = \frac{e^{ix}+e^{-ix}}{2}, \qquad \tan x = \frac{\sin x}{\cos x},$$

$$\cot x = \frac{1}{\tan x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x},$$

$$\sinh x = \frac{e^x-e^{-x}}{2}, \quad \cosh x = \frac{e^x+e^{-x}}{2}, \quad \tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x}, \quad \operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\sinh x}.$$

log undoes e^x:

$$\log(e^x) = x \quad \text{and} \quad e^{\log x} = x.$$