620-295 Real Analysis with applications

Assignment 1: Due 7 August 2009

Lecturer: Arun Ram
Department of Mathematics and Statistics
University of Melbourne
Parkville VIC 3010 Australia
aram@unimelb.edu.au

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1. Define the following sets and give examples of elements of each:
   (a) the set of rational numbers,
   (b) the set of real numbers,
   (c) the set of complex numbers.
2. Let $\frac{a}{b}, \frac{c}{d}, \frac{e}{f} \in \mathbb{Q}$. Show that $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$.
3. State and prove the Pythagorean Theorem.
4. Compute and graph the following:
   (a) $\frac{-15+i}{4+2i}$,
   (b) $\left(27^{1/3}\right)^4$,
   (c) $27^{(4+1/3)}$.
5. Let $z = x + iy$ with $x, y \in \mathbb{R}$. Compute and graph $\left|\frac{(3+4i)(-1+2i)}{(-1-i)(3-i)}\right|$.
6. Define the following and give examples:
   (a) injective,
   (b) surjective,
   (c) composition of functions,
   (d) abelian group.
7. Let $D : \mathbb{Q}[x] \longrightarrow \mathbb{Q}[x]$ be a function such that
   (a) If $f, g \in \mathbb{Q}[x]$ then $D(f+g) = D(f) + D(g)$
   (b) If $c \in \mathbb{Q}$ and $f \in \mathbb{Q}[x]$ then $D(cf) = cD(f)$,
   (c) If $f, g \in \mathbb{Q}[x]$ then $D(fg) = fD(g) + D(f)g$, and
   (d) $D(x) = 1$.
   Compute $D(x^n)$, for $n \in \mathbb{Z}_{\geq 0}$.
8. Write $\frac{1-x^n}{1-x}$ as an element of $\mathbb{Q}[x]$.