MATH 221: Calculus and Analytic Geometry
Prof. Ram, Fall 2006

HOMEWORK 10
DUE November 13, 2006

Problem A. Integrals with exponential functions and logarithms.

(1) $\displaystyle\int e^{2x-1}\,dx$

(2) $\displaystyle\int e^{1-3x}\,dx$

(3) $\displaystyle\int 3^{2-3x}\,dx$

(4) $\displaystyle\int \frac{1}{x\ln x}\,dx$

(5) $\displaystyle\int \frac{\ln(x^2)}{x}\,dx$

(6) $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

(7) $\displaystyle\int \frac{1}{x^2}e^{-1/x}\,dx$

(8) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

(9) $\displaystyle\int \frac{e^{2x}}{e^{2x}-2}\,dx$

(10) $\displaystyle\int \frac{\sqrt{2+\ln x}}{x}\,dx$

(11) $\displaystyle\int \frac{(x+1)(x+\ln x)^2}{x}\,dx$

(12) $\displaystyle\int \sqrt{e^x-1}\,dx$

Problem B. Definite integrals.

(1) $\displaystyle\int_{-2}^{4} (3x - 5)\ dx$

(2) $\displaystyle\int_{1}^{2} x^{-2}\ dx$

(3) $\displaystyle\int_{0}^{1} (1 - 2x - 3x^2)\ dx$

(4) $\displaystyle\int_{1}^{2} (5x^2 - 4x + 3)\ dx$

(5) $\displaystyle\int_{-3}^{0} (5y^4 - 6y^2 + 14)\ dy$

(6) $\displaystyle\int_{0}^{1} (y^9 - 2y^5 + 3y)\ dy$

(7) $\displaystyle\int_{0}^{4} \sqrt{x}\ dx$

(8) $\displaystyle\int_{0}^{1} x^{3/7}\ dx$

(9) $\displaystyle\int_{1}^{3} \left(\frac{1}{t^2} - \frac{1}{t^4}\right)\ dt$

(10) $\displaystyle\int_{1}^{2} \frac{t^6 - t^2}{t^4}\ dt$

(11) $\displaystyle\int_{1}^{2} \frac{x^2 + 1}{\sqrt{x}}\ dx$

(12) $\displaystyle\int_{0}^{2} (x^3 - 1)^2\ dx$

(13) $\displaystyle\int_{0}^{1} u(\sqrt{u} + \sqrt[3]{u})\ du$

(14) $\displaystyle\int_{-1}^{1} \frac{3}{t^4}\ dt$

(15) $\displaystyle\int_1^2 (x + 1/x)^2 \; dx$

(16) $\displaystyle\int_3^3 \sqrt{x^5 + 2} \; dx$

(17) $\displaystyle\int_{-4}^2 \frac{2}{x^6} \; dx$

(18) $\displaystyle\int_1^{-1} (x - 1)(3x + 2) \; dx$

(29) $\displaystyle\int_1^4 (\sqrt{t} - 2/\sqrt{t}) \; dt$

(20) $\displaystyle\int_1^8 \left(\sqrt[3]{r} + \frac{1}{\sqrt[3]{r}} \right) \; dr$

(21) $\displaystyle\int_{-1}^0 (x + 1)^3 \; dx$

(22) $\displaystyle\int_{-5}^{-2} \frac{x^4 - 1}{x^2 + 1} \; dx$

(23) $\displaystyle\int_1^e \frac{x^2 + x + 1}{x} \; dx$

(24) $\displaystyle\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 \; dx$

(25) $\displaystyle\int_0^1 \left(\sqrt[4]{x^5} + \sqrt[5]{x^4} \right) \; dx$

(26) $\displaystyle\int_1^8 \frac{x - 1}{\sqrt[3]{x^2}} \; dx$

Problem C. Definite integrals with trigonometric functions.

(1) $\displaystyle\int_{\pi/4}^{\pi/3} \sin t \; dt$

(2) $\displaystyle\int_0^{\pi/2} (\cos\theta + 2\sin\theta) \; d\theta$

(3) $\displaystyle\int_{\pi/2}^{\pi} \sec x \tan x \; dx$

(4) $\displaystyle\int_{\pi/3}^{\pi/2} \csc x \cot x \; dx$

(5) $\displaystyle\int_{\pi/6}^{\pi/3} \csc^2 \theta \; d\theta$

(6) $\displaystyle\int_{\pi/4}^{\pi} \sec^2 \theta \; d\theta$

(7) $\displaystyle\int_{1}^{\sqrt{3}} \frac{6}{1+x^2} \; dx$

(8) $\displaystyle\int_{0}^{0.5} \frac{dx}{\sqrt{1-x^2}} \; dx$

Problem D. Definite integrals with other functions.

(1) $\displaystyle\int_{4}^{8} 1/x \; dx$

(2) $\displaystyle\int_{\ln 3}^{\ln 6} 8e^x \; dx$

(3) $\displaystyle\int_{8}^{9} 2^t \; dt$

(4) $\displaystyle\int_{-e^2}^{-e} \frac{3}{x} \; dx$

(5) $\displaystyle\int_{-2}^{3} |x^2-1| \; dx$

(6) $\displaystyle\int_{-1}^{2} |x-x^2| \; dx$

(7) $\displaystyle\int_{-1}^{2} (x-2|x|) \; dx$

(8) $\displaystyle\int_{0}^{2} (x^2-|x-1|) \; dx$

(9) $\displaystyle\int_0^2 f(x)\,dx$ where $f(x) = \begin{cases} x^4, & \text{if } 0 \le x < 1, \\ x^5, & \text{if } 1 \le x \le 2. \end{cases}$

(10) $\displaystyle\int_{-\pi}^{\pi} f(x)\,dx$ where $f(x) = \begin{cases} x, & \text{if } -\pi \le x \le 0, \\ \sin x, & \text{if } 0 < x \le \pi. \end{cases}$

Problem E. The Fundamental Theorem of Calculus.

(1) What does $\displaystyle\int_a^b f(x)dx$ mean?

(2) How does one usually calculate $\displaystyle\int_a^b f(x)dx$? Give an example which shows that this method doesn't always work. Why doesn't it?

(3) Give an example which shows that $\displaystyle\int_a^b f(x)dx$ is not always the true area under $f(x)$ between a and b even if $f(x)$ is continuous between a and b.

(4) What is the Fundamental Theorem of Calculus?

(5) Let $f(x)$ be a function which is continuous and let $A(x)$ be the area under $f(x)$ from a to x. Compute the derivative of $A(x)$ by using limits.

(6) Why is the Fundamental Theorem of Calculus true? Explain carefully and thoroughly.

(7) Give an example which illustrates the Fundamental Theorem of Calculus. In order to do this compute an area by summing up the areas of tiny boxes and then show that applying the Fundamental Theorem of Calculus gives the same answer.

Problem F. Finding areas bounded by lines and a curve.

(1) Find the area of the region bounded by the curve $xy - 3x - 2y - 10 = 0$, the x-axis, and the lines $x = 3$ and $x = 4$.

(2) Find the area lying below the x-axis and above the parabola $y = 4x + x^2$.

(3) Graph the curve $y = 2\sqrt{9 - x^2}$ and determine the area enclosed between the curve and the x-axis.

(4) Find the area bounded by the curve $y = x(x-3)(x-5)$, the x-axis and the lines $x = 0$ and $x = 5$.

(5) Find the area enclosed between the curve $y = \sin 2x$, $0 \le x \le \pi/4$ and the axes.

(6) Find the area enclosed between the curve $y = \cos 2x$, $0 \le x \le \pi/4$ and the axes.

(7) Find the area enclosed between the curve $y = 3\cos x$, $0 \le x \le \pi/2$ and the axes.

(8) Show that the ratio of the areas under the curves $y = \sin x$ and $y = \sin 2x$ between the lines $x = 0$ and $x = \pi/3$ is $2/3$.

(9) Find the area enclosed between the curve $y = \cos 3x$, $0 \le x \le \pi/6$ and the axes.

(10) Find the area enclosed between the curve $y = \tan^2 x$, $0 \le x \le \pi/4$ and the axes.

(11) Find the area enclosed between the curve $y = \csc^2 x$, $0 \le x \le \pi/4$ and the axes.

(12) Compare the areas under the curves $y = \cos^2 x$ and $y = \sin^2 x$ between $x = 0$ and $x = \pi$.

(13) Graph the curve $y = x/\pi + 2\sin^2 x$ and find the area between the x-axis, the curve and the lines $x = 0$ and $x = \pi$.

(14) Find the area bounded by $y = \sin x$ and the x-axis between $x = 0$ and $x = 2\pi$.

Problem G. Areas between curves.

(1) Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $y = 2x$.

(2) Find the area bounded by the curve $y = x(2 - x)$ and the line $x = 2y$.

(3) Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

(4) Calculate the area of the region bounded by the parabolas $y = x^2$ and $x = y^2$.

(5) Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

(6) Find the area of the region bounded by the curves $y = \sqrt{x}$ and $y = x$.

(7) Find the area of the part of the first quadrant which is between the parabola $y^2 = 3x$ and the circle $x^2 + y^2 - 6x = 0$.

(8) Find the area of the region between the curves $y^2 = 4x$ and $x = 3$.

(9) Use integration to find the area of the triangular region bounded by the lines $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

(10) Find the area bounded by the parabola $x^2 - 2 = y$ and the line $x + y = 0$.

(11) Find the area bounded by the curves $y = 3x - x^2$ and $y = x^2 - x$.

(12) Graph the curve $y = (1/2)x^2 + 1$ and the straight line $y = x + 1$ and find the area between the curve and the line.

(13) Find the area of the region between the parabolas $4y^2 = 9x$ and $3x^2 = 16y$.

(14) Find the area of the region between the curves $x^2 + y^2 = 2$ and $x = y^2$.

(15) Find the area of the region between the curves $y = x^2$ and $x^2 + 4(y - 1) = 0$.

(16) Find the area of the region between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

(17) Find the area of the region enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$.

(18) Find the area between the parabolas $y = 4ax$ and $y^2 = 4ay$.

(19) Find the area of the region between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.

(20) Find the area bounded by the curves $y = x$ and $y = x^3$.

(21) Graph $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \pi/2$ and find the area enclosed by them and the x-axis.