620-295 Real Analysis with Applications

Assignment 3: Due 5pm on 4 September

Lecturer: Arun Ram
Department of Mathematics and Statistics
University of Melbourne
Parkville VIC 3010 Australia
aram@unimelb.edu.au

Due 5pm on 4 September in the appropriate assignment box on the ground floor of Richard Berry.

1. Define the following and give an example for each:
   (a) cardinality,
   (b) finite,
   (c) infinite,
   (d) countable,
   (e) uncountable.

2. Prove that $\mathrm{Card}(\mathbb{Z}_{>0}) \neq \mathrm{Card}(\mathbb{R})$.

3. Define the following and give an example for each:
   (a) sequence,
   (b) converges (for a sequence),
   (c) diverges (for a sequence),
   (d) limit (of a sequence),
   (e) sup (of a sequence),
   (f) inf (of a sequence),
   (g) lim sup (of a sequence),
   (h) lim inf (of a sequence),
   (i) bounded (for a sequence),
   (j) increasing (for a sequence),
   (k) decreasing (for a sequence),
   (l) monotone (for a sequence),
   (m) Cauchy sequence.

4. Give an example of a sequence (a_n) such that none of $\inf a_n$, $\liminf a_n$, $\limsup a_n$, and sup a_n are equal.

5. Find the power series expansions and the radius of convergence of e^x, $\log(1+x)$, $\frac{1}{1-x}$, $(1+x)^{1/2}$, $\arctan x$, and $\sinh x$.

6. Let $r \in \mathbb{R}$ with $0 < r < 1$. Find $\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n$, and explain why this limit is important to everyone with a credit card.

7. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

8. Let $r \in \mathbb{R}$. Find (with proof) $\sum_{n=1}^{\infty} r^n$.

9. Show that the alternating harmonic series for $\arctan 1$ is conditionally convergent but not absolutely convergent. Explain how to rearrange it so that its sum is 301.