620-295 Real Analysis with Applications

Assignment 2: Due 5pm on 21 August

Lecturer: Arun Ram
Department of Mathematics and Statistics
University of Melbourne
Parkville VIC 3010 Australia
aram@unimelb.edu.au

Due 5pm on 21 August in the appropriate assignment box on the ground floor of Richard Berry.

1. Let $f : S \to T$ be a function. Show that the inverse function to f exists if and only if f is bijective.

2. Add up the positive integers from 1 to 100. Then add up the squares 1^2 to 100^2.

3. Let S be a set with an associative operation with identity. Show that the identity is unique. (This tells us that any commutative monoid has only one heart.)

4. Let S be a set with an associative operation with identity. Let $s \in S$ and assume that s has an inverse in S. Show that the inverse of s is unique. (This tells us that any element of an abelian group has only one mate.)

5. Let S be a ring. Show that if $s \in S$ then $s \cdot 0 = 0$.

6. Prove that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)$.

7. Define the following and give an example for each:
   (a) order,
   (b) maximum,
   (c) minimum,
   (d) upper bound,
   (e) lower bound,
   (f) bounded above,
   (g) bounded below,
   (j) supremum,
   (k) infimum,

8. Prove that if $n \in \mathbb{Z}_{>0}$ then $x - y$ is a factor of $x^n - y^n$.

9. For each of the following subsets of $\mathbb{R}$ find the maximum, the minimum, an upper bound, a lower bound, the supremum, and the infimum:
   (a) $\{2^{-m} - 3^n \mid m, n \in \mathbb{Z}_{\geq 0}\}$,
   (b) $\{x \in \mathbb{R} \mid x^3 - 4x < 0\}$,
   (c) $\{1 + x^2 \mid x \in \mathbb{R}\}$,

10. What is the triangle inequality and how do you justify it?