12.05.2025 ①
Calculus Lect
A. Ram

6.16 Suppose $\frac{dy}{dx} = y - \frac{y^2}{4}$. Solve for y.
6.14
6.22 If $\frac{dy}{dx} = 0$ then $0 = y - \frac{y^2}{4} = y\left(1 - \frac{y}{4}\right)$ and

$y = 0$ or $y = 4$ (constant solutions).

Otherwise,

$$\frac{dy}{dx} = \frac{4y - y^2}{4} = \frac{y(4-y)}{4} \quad \text{and}$$

$$\frac{4}{y(4-y)}\frac{dy}{dx} = 1. \quad \text{So} \quad \frac{(4-y+y)}{y(4-y)}\frac{dy}{dx} = 1.$$

So $$\left(\frac{1}{y} + \frac{1}{4-y}\right)\frac{dy}{dx} = 1.$$

So $$\int\left(\frac{1}{y} + \frac{1}{4-y}\right)\frac{dy}{dx}\,dx = \int 1\cdot dx$$

So $\log(y) - \log(4-y) = x + c$, where c is a constant.

So $\log\left(\frac{y}{4-y}\right) = x + c$. and $\frac{y}{4-y} = e^{x+c} = e^c e^x$.

So $\frac{y}{4-y} = Ce^x$ where C is a constant.

So $y = (4-y)Ce^x = 4Ce^x - yCe^x$.

So $y(1+Ce^x) = 4Ce^x$. and $y = \frac{4Ce^x}{1+Ce^x}$.

So $y = \frac{4}{\frac{1}{C}e^{-x}+1} = \frac{4}{Ke^{-x}+1}$, where K is a constant.

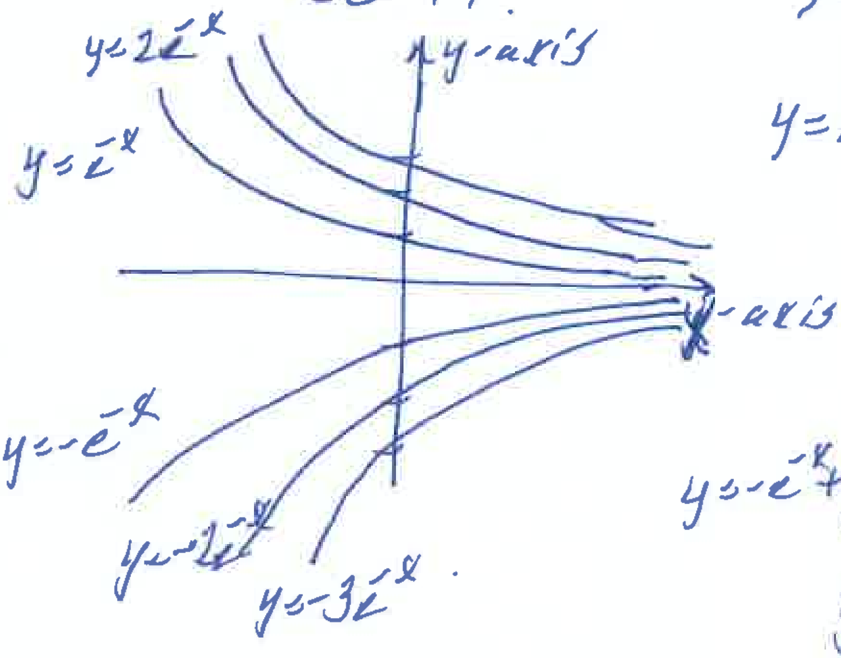
6.11 — 12.05.2015, Calculus Lect., A. Ram — ②

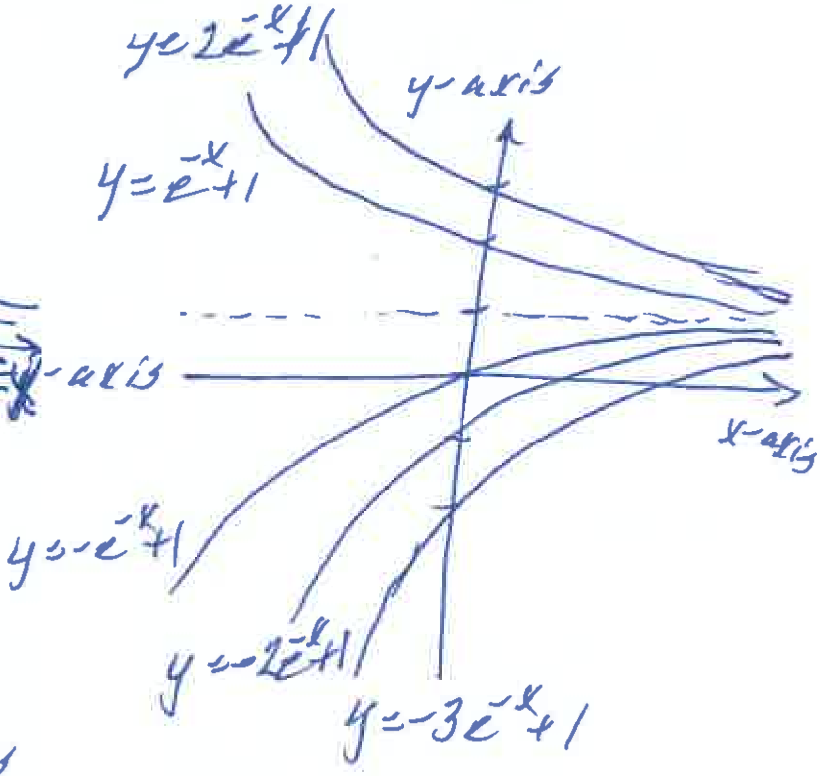
Suppose $\frac{dP}{dt} = P - \frac{P^2}{4}$ with $P(0) = 1$.

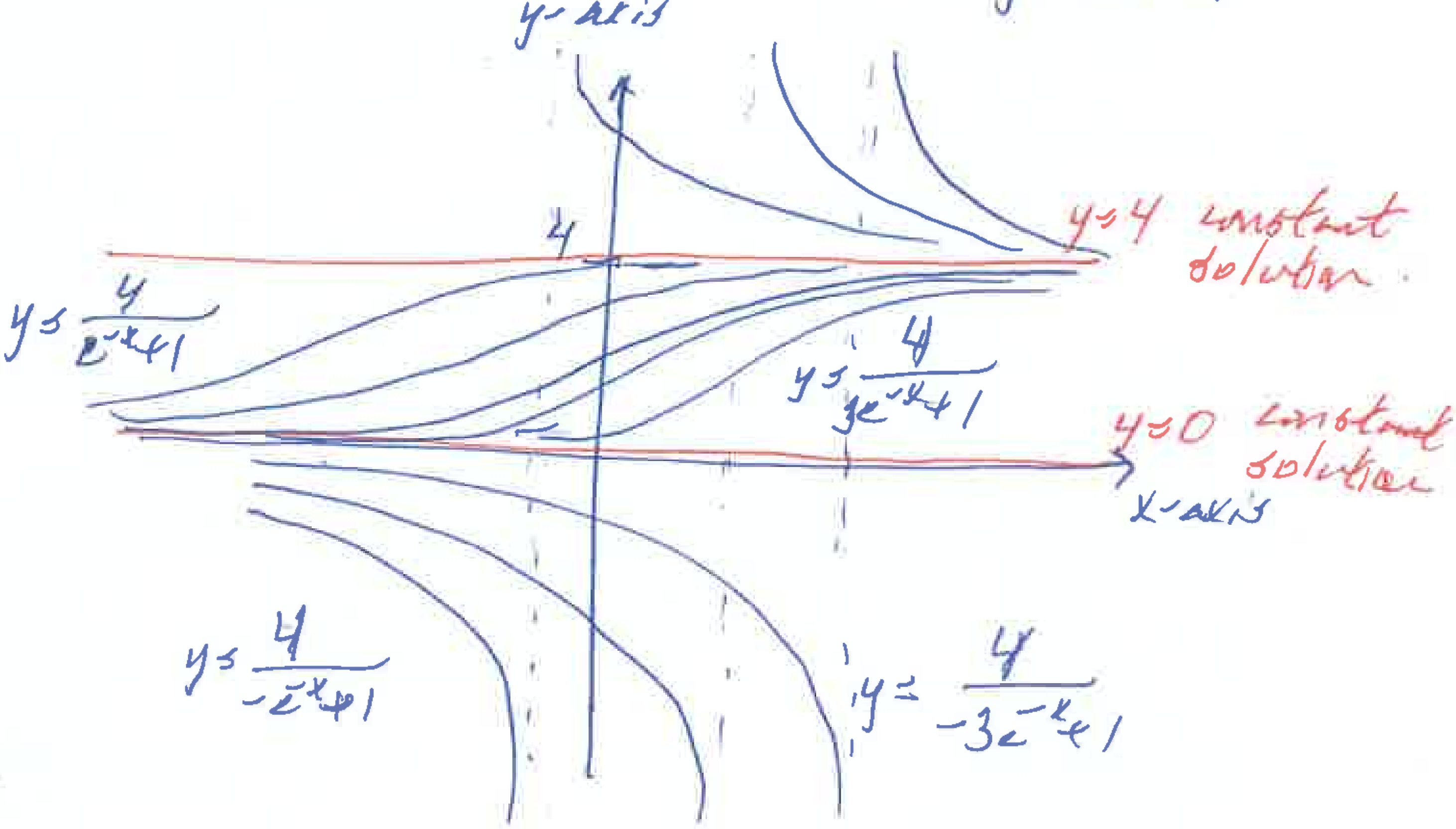
Then $1 = P(0) = \frac{4}{Ke^{-0}+1} = \frac{4}{K+1}$

and $K+1 = 4$ so that $K = 3$.

So $P = \frac{4}{3e^{-t}+1}$.







14.05.2025 ③
Calculus Lect
A. Ram

Models of change

6.21 The rate of growth of a mouse population is proportional to the current number of mice present.

$$\frac{dM}{dt} = 0.2M$$

There are initially 50 mice. $M(0) = 50$.

When will the mouse population reach 500?

Find t such that $M(t) = 500$.

Since $\frac{dM}{dt} = 0.2M$ then $\frac{1}{M}\frac{dM}{dt} = 0.2$.

So $\int \frac{1}{M}\frac{dM}{dt}\,dt = \int 0.2\,dt$. Then $\log|M| = 0.2t + c$.

So $M = e^{0.2t+c} = e^c e^{0.2t} = Ce^{0.2t}$, where C is a constant.

Since $M(0) = 50 = Ce^{0.2\cdot 0} = Ce^0 = C$ then

$$M = 50e^{0.2t}.$$

If $500 = M = 50e^{\frac{1}{5}t}$ then $10 = e^{\frac{1}{5}t}$.

So $\log(10) = \frac{1}{5}t$ and $t = 5\log(10)$,

when $M = 500$.

Newton's law of cooling

$\frac{dT}{dt} = -k(T - T_s)$, where T_s = surrounding temperature and k is a constant depending on the material.

6.23 A loaf of bread is placed in a freezer whose temperature is a constant $-15°C$. The temperature T of the bread is initially $20°C$ and it takes 20 minutes for it to drop to $10°C$. How long will it take to reach $0°C$.

$T_s = -15$, $T(0) = 20$ and $T(20) = 10$

So $\frac{dT}{dt} = -k(T - (-15)) = -k(T+15)$.

So $\frac{1}{(T+15)} \frac{dT}{dt} = -k$ and $\int \frac{1}{(T+15)} \frac{dT}{dt}\, dt = \int -k\, dt$.

So $\log(T+15) = -kt + c$, where c is a constant.

So $T + 15 = e^{-kt+c} = e^c e^{-kt} = Ce^{-kt}$, where C is a constant.

So $20 + 15 = T(0) + 15 = Ce^{-k \cdot 0} = Ce^0 = C$ and

$T + 15 = 35e^{-kt}$.

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Then $10+15 = T(20)+15 = 35e^{-k\cdot 20}$

So $\frac{25}{35} = e^{-k\cdot 20}$ and $\log\left(\frac{25}{35}\right) = -k\,20$

So $k = \frac{-1}{20}\log\left(\frac{5}{7}\right)$.

So
$$T(t) = 35\,e^{\frac{1}{20}\log\left(\frac{5}{7}\right)t}.$$

If $T(t) = 0$ then $15+0 = 35\,e^{\frac{1}{20}\log\left(\frac{5}{7}\right)t}$

and $\frac{15}{35} = e^{\frac{1}{20}\log\left(\frac{5}{7}\right)t}$. So $\log\left(\frac{15}{35}\right) = \frac{1}{20}\log\left(\frac{5}{7}\right)t$

and $t = \frac{\log\left(\frac{3}{7}\right)}{\log\left(\frac{5}{7}\right)}\cdot 20 = 20\,\frac{\log\left(\frac{3}{7}\right)}{\log\left(\frac{5}{7}\right)}$.