09.06.2025 ③

Calculus Lect

A. Ram

6.11 Suppose $\frac{dy}{dx} = -6xy^2$. Solve for y.

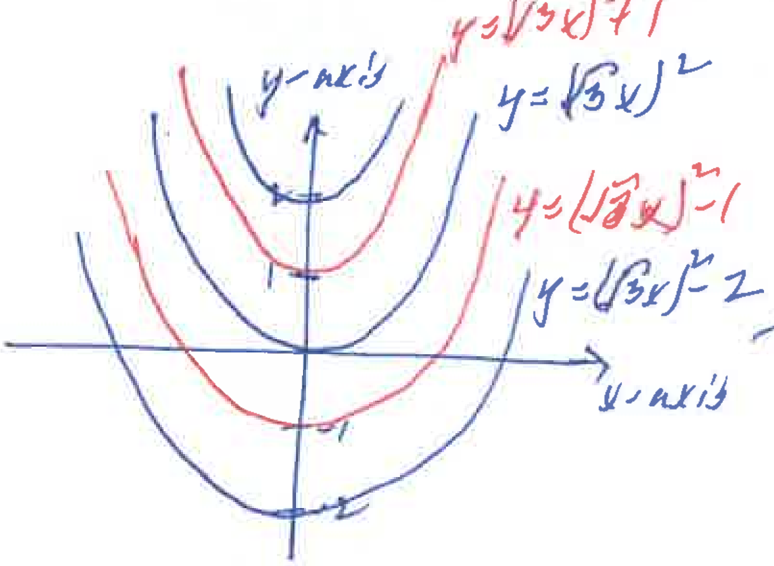
If $\frac{dy}{dx} = 0$ then $0 = y^2$ and $y = 0$ (constant solution)

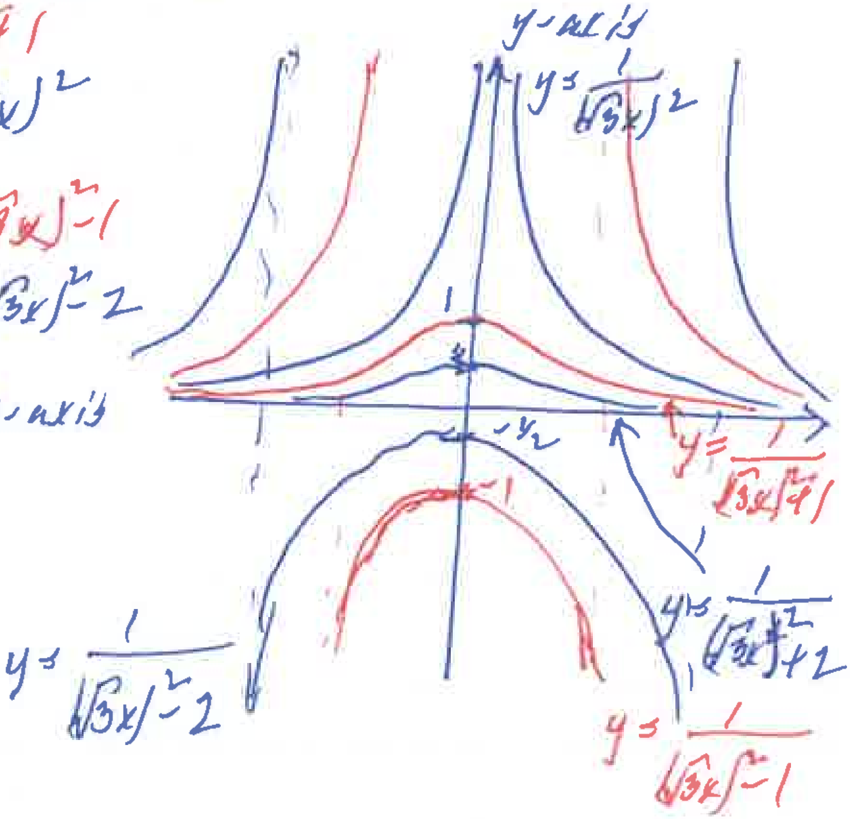
Otherwise $\frac{-1}{y^2}\frac{dy}{dx} = 6x$ and

$$\int -y^{-2}\frac{dy}{dx}\,dx = \int 6x\,dx.$$

So $y^{-1} = 3x^2 + c$ where c is a constant.

So $$y = \frac{1}{3x^2+c} = \frac{1}{(\sqrt{3}x)^2+c}$$





Calculus Lect.
A. Ram

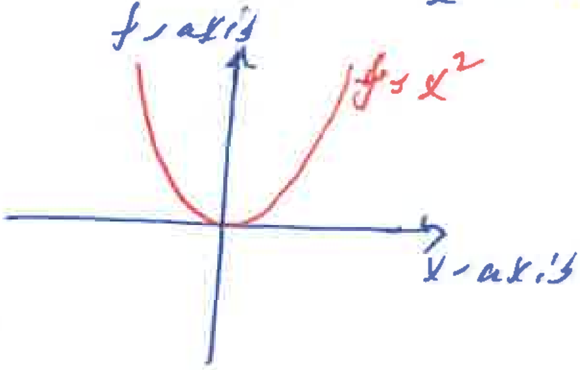
<u>6.8</u> Suppose $f'(x) + \frac{1}{x} f(x) = 3x$. Solve for f.

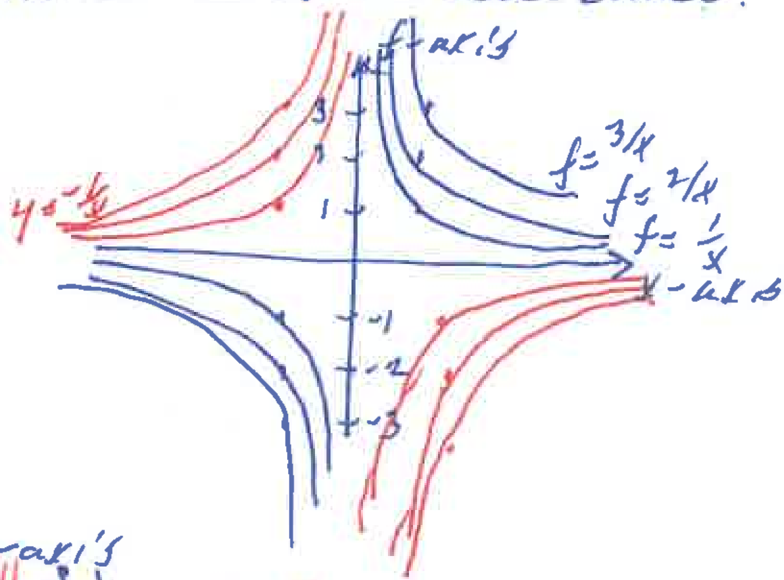
Since $x f'(x) + 1 \cdot f(x) = 3x^2$ then

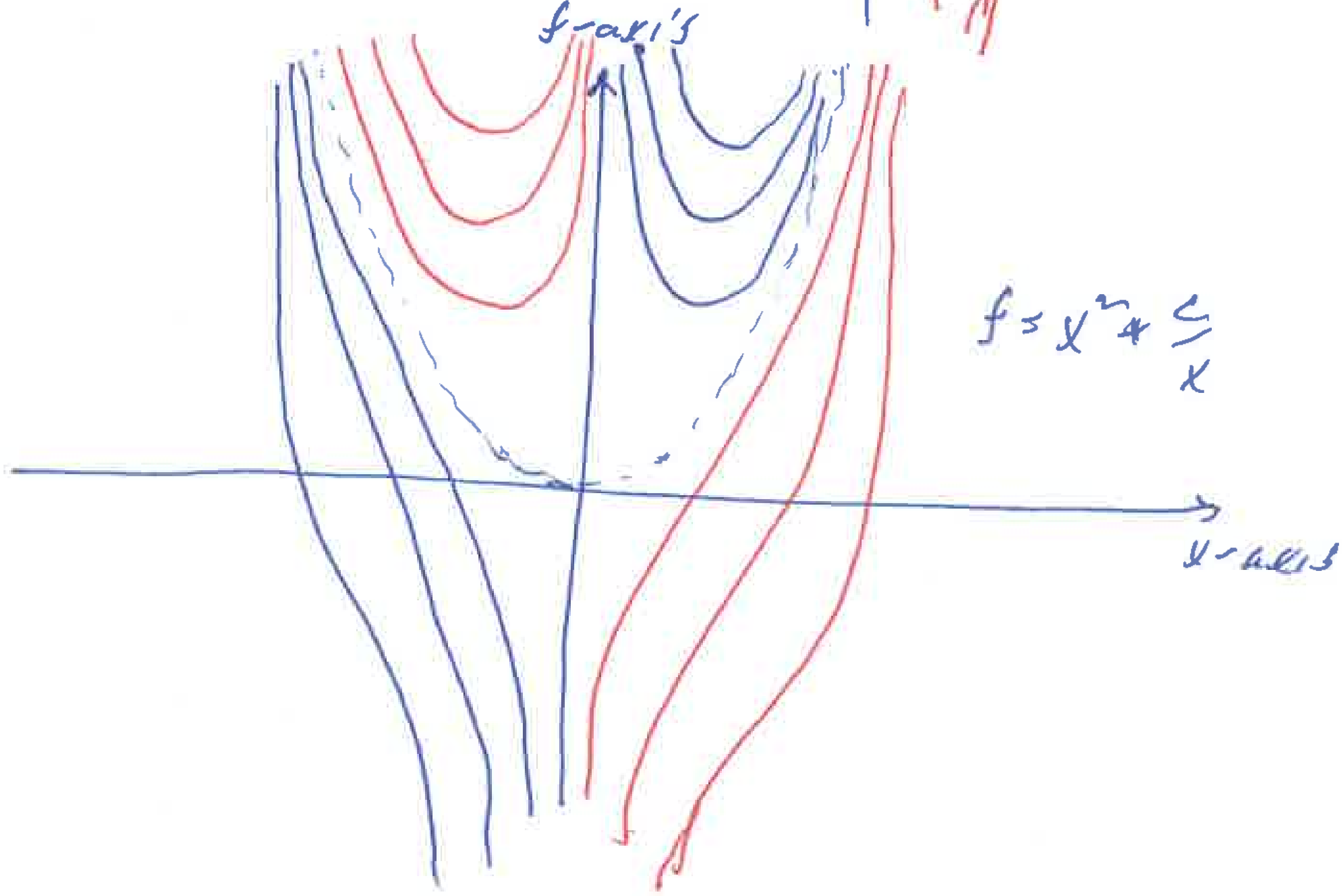
$$\frac{d(xf)}{dx} = 3x^2 \quad \text{and} \quad xf = x^3 + c,$$

where c is a constant.

So $f = x^2 + \frac{c}{x}$, where c is a constant.







12.05.2015 ④

CalculusLect

A.Ram

6.20 Suppose $\frac{dy}{dx} = y^{\frac{1}{3}}$. Solve for y.

If $\frac{dy}{dx} = 0$ then $y = 0$ (constant solution)

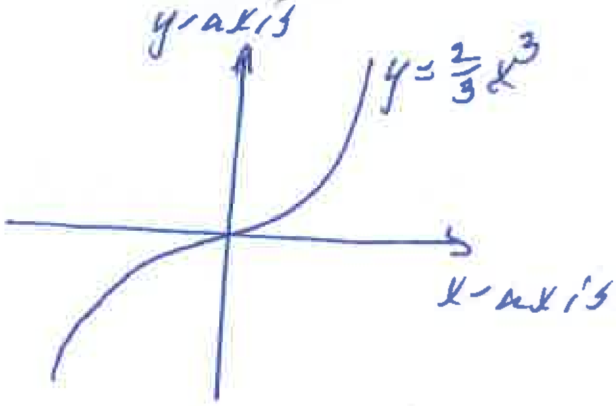
Otherwise

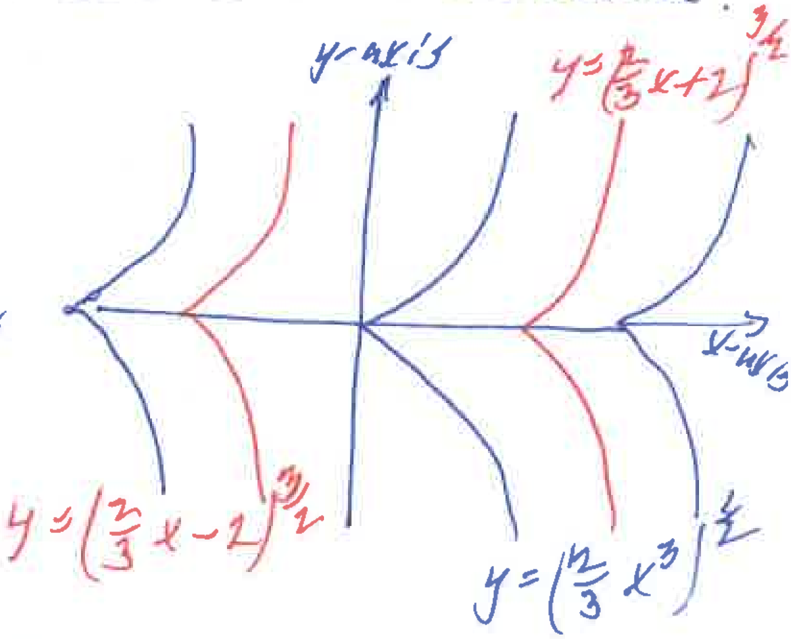
$$y^{-1/3}\frac{dy}{dx} = 1 \quad \text{and} \quad \int y^{-1/3}\frac{dy}{dx}\,dx = \int 1\cdot dx.$$

So $\frac{3}{2}y^{2/3} = x + c$ and $y^{2/3} = \frac{2}{3}x + \frac{2}{3}c$,

where c is a constant.

So $\quad y^2 = \left(\frac{2}{3}x + C\right)^3$, where C is a constant.





12.05.2025 ①
Calculus Lect
A. Ram

6.16 Suppose $\frac{dy}{dx} = y - \frac{y^2}{4}$. Solve for y.
6.19
6.22 If $\frac{dy}{dx} = 0$ then $0 = y - \frac{y^2}{4} = y\left(1 - \frac{y}{4}\right)$ and

$y = 0$ or $y = 4$ (constant solutions).

Otherwise,

$$\frac{dy}{dx} = \frac{4y - y^2}{4} = \frac{y(4-y)}{4} \quad \text{and}$$

$$\frac{4}{y(4-y)}\frac{dy}{dx} = 1. \quad \text{So} \quad \frac{(4-y+y)}{y(4-y)}\frac{dy}{dx} = 1.$$

So $\left(\frac{1}{y} + \frac{1}{4-y}\right)\frac{dy}{dx} = 1.$

So $\int\left(\frac{1}{y} + \frac{1}{4-y}\right)\frac{dy}{dx}\,dx = \int 1 \cdot dx$

So $\log(y) - \log(4-y) = x + c$, where c is a constant.

So $\log\left(\frac{y}{4-y}\right) = x + c$. and $\frac{y}{4-y} = e^{x+c} = e^c e^x$.

So $\frac{y}{4-y} = Ce^x$ where C is a constant.

So $y = (4-y)Ce^x = 4Ce^x - yCe^x$.

So $y(1+Ce^x) = 4Ce^x$. and $y = \frac{4Ce^x}{1+Ce^x}$.

So $y = \frac{4}{\frac{1}{C}e^{-x}+1} = \frac{4}{Ke^{-x}+1}$, where K is a constant.

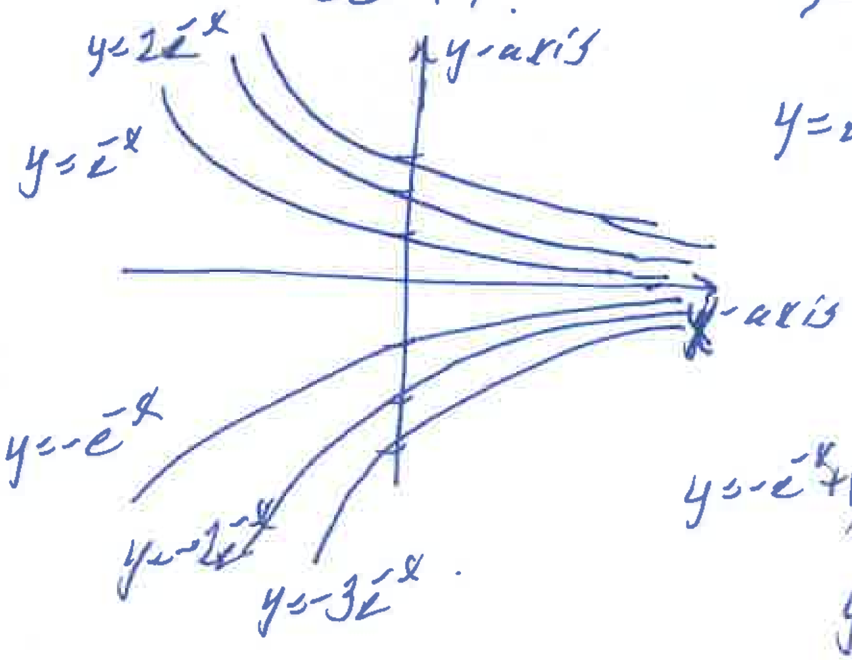
6.11 12.05.2025 Calculus Lect. A. Ram ②

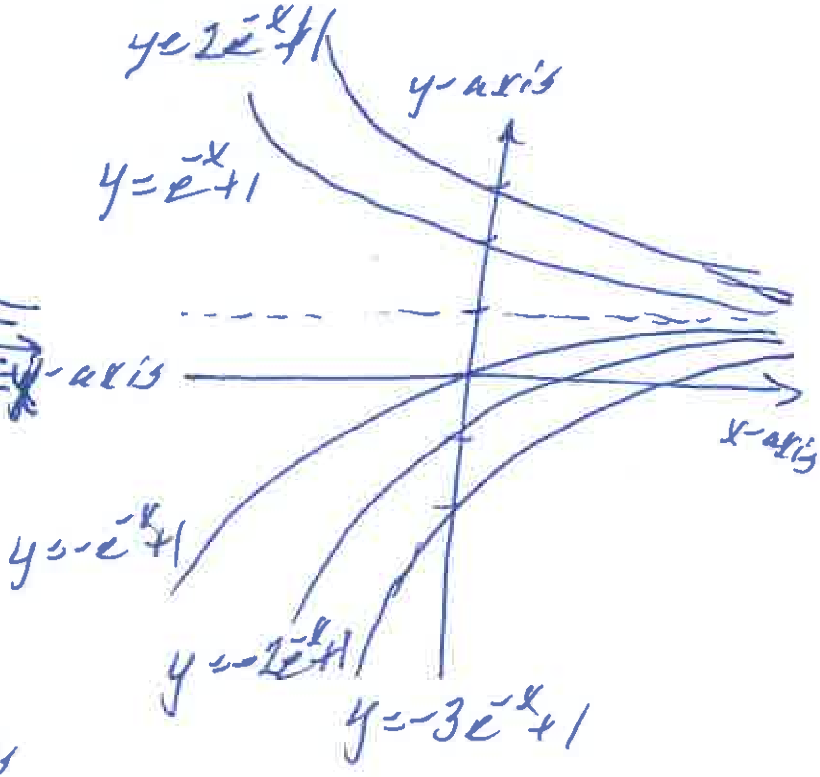
Suppose $\frac{dP}{dt} = P - \frac{P^2}{4}$ with $P(0) = 1$.

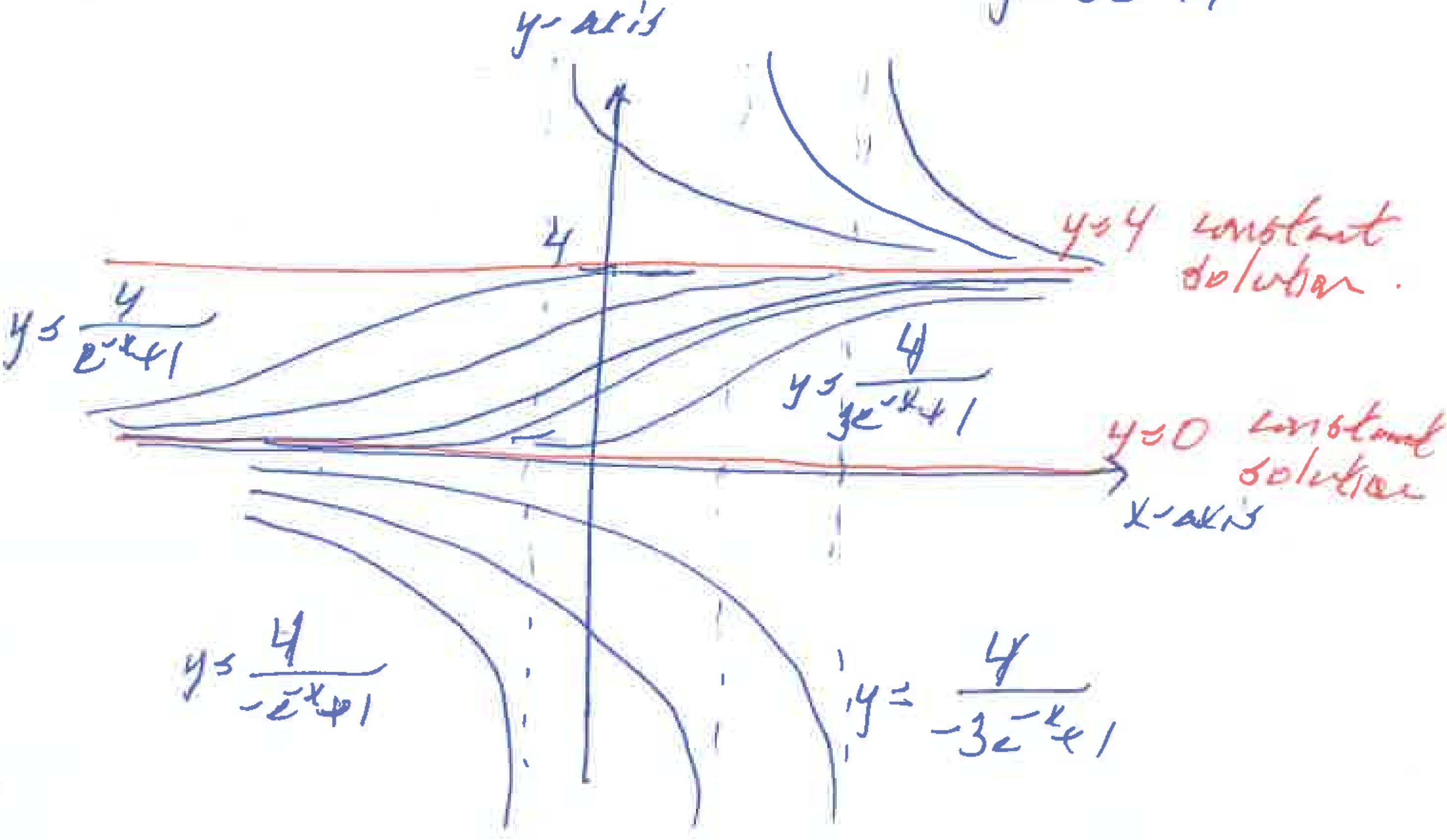
Then $1 = P(0) = \frac{4}{Ke^{-0}+1} = \frac{4}{K+1}$

and $K+1 = 4$ so that $K = 3$.

So $P = \frac{4}{3e^{-t}+1}$.







12.05.2025
Calculus lect. 5
A. Ram

6.4 Verify that

$y = c_1 e^{3x} + c_2 e^{-5x}$, where c_1 and c_2 are arbitrary constants

are <u>all</u> solutions of

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 15y = 0$$

Since

$$y = c_1 e^{3x} + c_2 e^{-5x}$$

$$\frac{dy}{dx} = 3c_1 e^{3x} - 5e^{-5x}$$

$$\frac{d^2y}{dx^2} = 9c_1 e^{3x} + 25e^{-5x}$$

then

$$-15y = -15c_1 e^{3x} - 15c_2 e^{-5x}$$

$$2\frac{dy}{dx} = 6c_1 e^{3x} - 10e^{-5x}$$

$$\frac{d^2y}{dx^2} = 9c_1 e^{3x} + 25e^{-5x}.$$

So $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 15y = 0.$$