05.05.2025 ④
Calculus Lect
A. Ram

$$\int_{x=0}^{x=4} (y_{top} - y_{bottom})\,dx = \int_{x=0}^{x=4} (\sqrt{x} - \tfrac{1}{2}x)\,dx$$

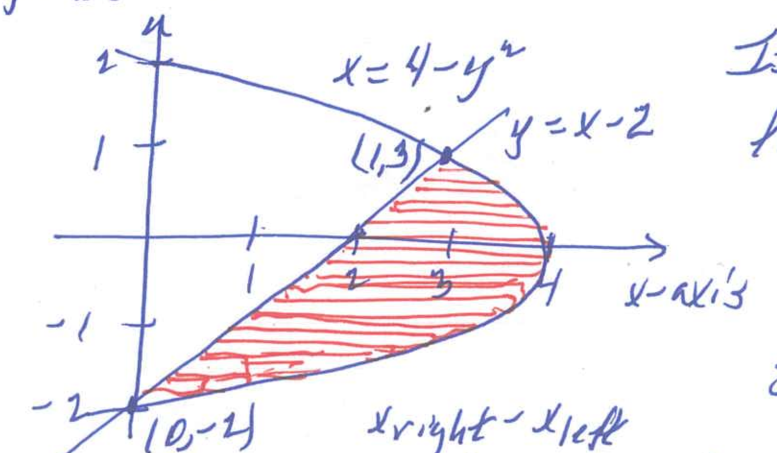
$$= \int_{x=0}^{x=4} (x^{1/2} - \tfrac{1}{2}x)\,dx = \tfrac{2}{3}x^{3/2} - \tfrac{1}{2}\cdot\tfrac{1}{2}x^2\Big]_{x=0}^{x=4}$$

$$= \left(\tfrac{2}{3}\cdot 4^{3/2} - \tfrac{1}{4}\cdot 4^2\right) - \left(\tfrac{2}{3}\cdot 0^{3/2} - \tfrac{1}{4}\cdot 0^2\right) = \tfrac{2}{3}\cdot 2^3 - 4$$

$$= \tfrac{16}{3} - \tfrac{12}{3} = \tfrac{4}{3}.$$

5.6 Find the area enclosed by

$y = x - 2$ and $x = 4 - y^2$



If $x = 4 - y^2$ and $y = x - 2$ then

$y = 4 - y^2 - 2 = 2 - y^2$

So $y^2 + y - 2 = 0$.

So $(y+2)(y-1) = 0$.

Add slices dy from $y = -2$ to $y = 1$.

So
$$\int_{y=-2}^{y=1} (x_{right} - x_{left})\,dy = \int_{y=-2}^{y=1} ((4 - y^2) - (y+2))\,dy$$

$$= \int_{y=-2}^{y=1} (-y^2 + 4 - y - 2)\,dy = \int_{y=-2}^{y=1} (-y^2 - y + 2)\,dy$$

$$= -\tfrac{1}{3}y^3 - \tfrac{1}{2}y^2 + 2y\Big]_{y=-2}^{y=1} = \left(-\tfrac{1}{3}\cdot 1^3 - \tfrac{1}{2}\cdot 1^2 + 2\cdot 1\right) - \left(-\tfrac{1}{3}(-2)^3 - \tfrac{1}{2}(-2)^2 + 2(-2)\right)$$

$$= -\tfrac{1}{3} - \tfrac{1}{2} + 2 - \tfrac{8}{3} + 2 + 4 = 8 - 3 - \tfrac{1}{2} = 5 - \tfrac{1}{2} = \tfrac{9}{2}$$

5.6 Add slices $\}\, y_{top} - y_{bottom}$ from $x=0$ to $x=3$

dx

and add slices $\}\, y_{top} - y_{bottom}$ for $x=3$ to $x=4$

dx

$$\int_{x=0}^{x=3} (y_{top} - y_{bottom})\,dx + \int_{x=3}^{x=4} (y_{top} - y_{bottom})\,dx$$

$$= \int_{x=0}^{x=3} \left((x-2) + \sqrt{4-x}\right) dx + \int_{x=3}^{x=4} \left(\sqrt{4-x} + \sqrt{4-x}\right) dx$$

$$= \int_{x=0}^{x=3} \left(x-2+(4-x)^{1/2}\right) dx + \int_{x=3}^{x=4} 2(4-x)^{1/2}\,dx$$

$$= \frac{x^2}{2} - 2x + \frac{-2}{3}(4-x)^{3/2}\Big]_{x=0}^{x=3} + 2\cdot\frac{-2}{3}(4-x)^{3/2}\Big]_{x=3}^{x=4}$$

$$= \left(\frac{3^2}{2} - 2\cdot 3 - \frac{2}{3}\,1^{3/2}\right) - \left(\frac{0^2}{2} - 2\cdot 0 - \frac{2}{3}\,4^{3/2}\right)$$

$$+ \left(-\frac{4}{3}\,0^{3/2} - -\frac{4}{3}\,1^{3/2}\right)$$

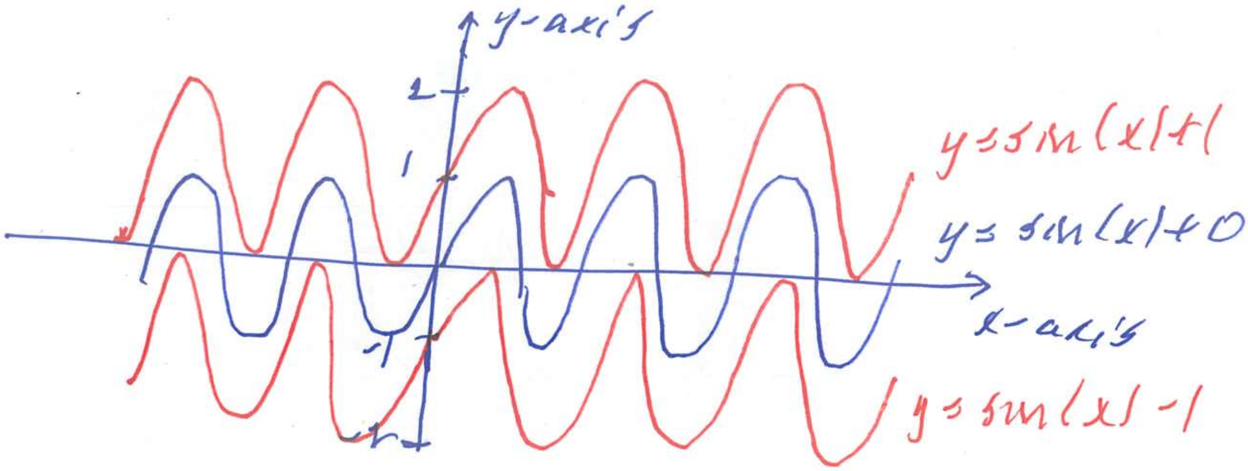
$$= \frac{9}{2} - 6 - \frac{2}{3} + \frac{2}{3}\cdot 8 + 0 + \frac{4}{3}$$

$$= -\frac{3}{2} + \frac{14}{3} + \frac{4}{3} = \frac{18}{3} - \frac{3}{2} = 6 - \frac{3}{2} = \frac{9}{2}.$$

Equations

6.5 Suppose $\frac{dy}{dx} = \cos(x)$. Solve for y.

Since
$$\int \frac{dy}{dx}\,dx = \int \cos(x)\,dx \text{ then } y = \sin(x) + c,$$
where c is a constant.



6.6 Suppose $\frac{dy}{dx} = \cos(x)$ with $y\left(\frac{\pi}{2}\right) = 3$.

Then
$$3 = y\left(\tfrac{\pi}{2}\right) = \sin\left(\tfrac{\pi}{2}\right) + c = 1 + c \text{ and } c = 2.$$

So $\quad y = \sin(x) + 2.$

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6.9 Suppose $\frac{dy}{dx} = y$. Solve for y.

If $\frac{dy}{dx} = 0$ then $y = 0$ (constant solution)

Otherwise $\frac{1}{y}\frac{dy}{dx} = 1$ and $\int \frac{1}{y}\frac{dy}{dx}\,dx = \int 1 \cdot dx$

So $\log(y) = x + c$, where c is a constant.

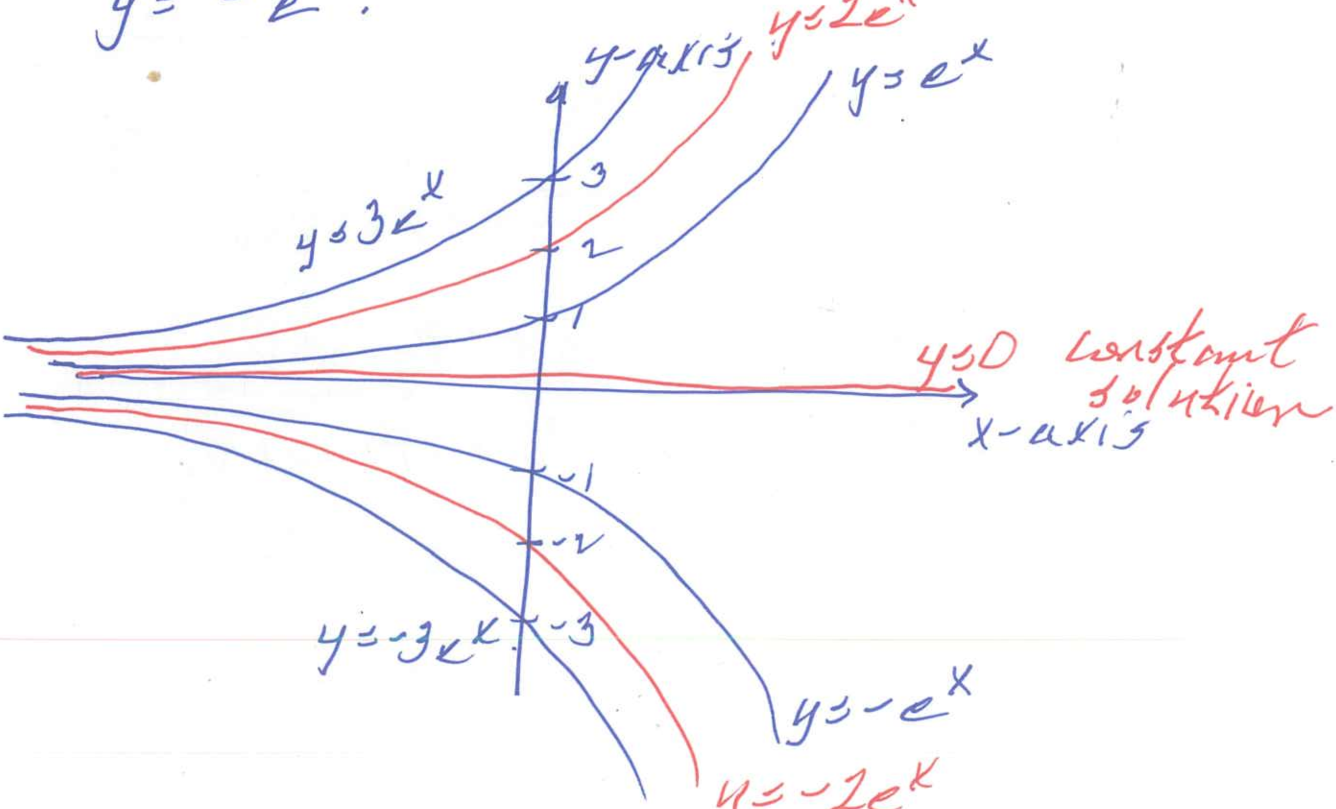
So $e^{\log(y)} = e^{x+c}$ and $y = e^c e^x$.

So $y = Ce^x$, where C is a constant

6.10 Suppose $\frac{dy}{dx} = y$ ~~with~~ with $y(0) = -1$.

Then $-1 = y(0) = Ce^0$ and $C = -1$.

So $y = -e^x$.



6.11 Suppose $\frac{dy}{dx} = -6xy^2$. Solve for y.

If $\frac{dy}{dx} = 0$ then $0 = y^2$ and $y = 0$ (constant solution)

Otherwise $\frac{-1}{y^2}\frac{dy}{dx} = 6x$ and

$$\int -y^{-2}\frac{dy}{dx}\,dx = \int 6x\,dx.$$

So $y^{-1} = 3x^2 + c$ where c is a constant.

So $y = \frac{1}{3x^2+c} = \frac{1}{(\sqrt{3}x)^2+c}$