02.05.2025
Calculus Lect. 21 ①

Partial fractions
Backwards of common denominator

A. Ram

$$\frac{1}{12} = \frac{1}{3\cdot 4} = \frac{4-3}{3\cdot 4} = \frac{1}{3} - \frac{1}{4}.$$

5.29 $$\int \frac{9x+1}{(x-3)(x+1)}\,dx = \int \frac{9(x+1)-8}{(x-3)(x+1)}\,dx$$

$$= \int \left(\frac{9}{x-3} - \frac{8}{(x-3)(x+1)}\right)dx = \int\left(\frac{9}{x-3} - 8\cdot\frac{1}{(x-3)(x+1)}\right)dx$$

$$= \int\left(\frac{9}{x-3} - \frac{8}{4}\cdot\frac{(x+1)-(x-3)}{(x-3)(x+1)}\right)dx$$

$$= \int\left(\frac{9}{x-3} - 2\left(\frac{1}{x-3} - \frac{1}{x+1}\right)\right)dx$$

$$= \int\left(\frac{7}{x-3} + \frac{2}{x+1}\right)dx = 7\log(x-3) + 2(x+1) + c,$$

where c is a constant.

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Calculus Lect 2
A. Ram

5.33 $\displaystyle\int \frac{2x^3-3x^2-8x+24}{x^2-4}\,dx$

$$= \int \frac{2x(x^2-4)-3(x^2-4)+12}{x^2-4}\,dx.$$

$$= \int \left(2x-3+12\cdot\frac{1}{x^2-4}\right)dx$$

$$= \int \left(2x-3+12\,\frac{1}{(x+2)(x-2)}\right)dx$$

$$= \int \left(2x-3+\frac{12}{4}\cdot\frac{(x+2)-(x-2)}{(x+2)(x-2)}\right)dx$$

$$= \int \left(2x-3+3\left(\frac{1}{x-2}-\frac{1}{x+2}\right)\right)dx$$

$$= \int \left(2x-3+\frac{3}{x-2}-\frac{3}{x+2}\right)dx$$

$$= x^2-3x+3\log(x-2)-3\log(x+2)+c,$$

where c is a constant.

5.8 Let $a \in \mathbb{R}$. Then

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+c,$$ where

c is a constant, because

$$\frac{d}{dx}\left(\frac{1}{a}\arctan\left(\frac{x}{a}\right)\right) = \frac{1}{a}\cdot\left(\frac{1}{\left(\frac{x}{a}\right)^2+1}\right)\cdot\frac{1}{a}$$

$$= \frac{1}{a^2\left(\left(\frac{x}{a}\right)^2+1\right)} = \frac{1}{x^2+a^2}.$$

5.26 and 5.25 and 5.13 $$\int \frac{x-2}{x^2+2x+2}\,dx = \int \frac{x-2}{(x+1)^2+1}\,dx$$

$$= \int \frac{(x+1)-3}{(x+1)^2+1}\,dx = \int\left(\frac{(x+1)}{(x+1)^2+1} - \frac{3}{(x+1)^2+1}\right)dx$$

$$= \int \frac{1}{2}\cdot\frac{1}{(x+1)^2+1}\cdot 2(x+1)\,dx - 3\int \frac{1}{(x+1)^2+1}\,dx$$

$$= \frac{1}{2}\log((x+1)^2+1) - 3\arctan(x+1)+c,$$

where c is a constant.

5.24
$$\int \frac{1}{2x^2+2x+4}\,dx = \frac{1}{2}\int \frac{1}{x^2+x+2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{x^2+x+\frac{1}{4}+\frac{7}{4}}\,dx = \frac{1}{2}\int \frac{1}{(x+\frac{1}{2})^2+\left(\frac{\sqrt{7}}{2}\right)^2}\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{\frac{\sqrt{7}}{2}}\arctan\left(\frac{\sqrt{7}}{2}\right)+c$$

$$= \frac{1}{\sqrt{7}}\arctan\left(\frac{\sqrt{7}}{2}\right)+c,$$ where c is a constant..

5.30
$$\int \frac{(3x^2-2x+1)}{(x+1)(x^2+2x+2)}\,dx = \int \frac{3(x+1)^2-8x-2}{(x+1)((x+1)^2+1)}\,dx$$

$$= \int \frac{3(x+1)^2-8(x+1)+6}{(x+1)((x+1)^2+1)}\,dx.$$

Let $u = x+1$
So $x = u-1$
and $\frac{dx}{du} = 1$.

$$= \int \frac{3u^2-8u+6}{u(u^2+1)}\,\frac{dx}{du}\,du$$

$$= \int\left(\frac{3u}{u^2+1} - \frac{8}{u^2+1} + 6\,\frac{1}{u(u^2+1)}\right)du$$

$$= \frac{3}{2}\int \frac{2u}{u^2+1}\,du - 8\int\frac{1}{u^2+1}\,du + 6\int \frac{u^2+1-u\cdot u}{u(u^2+1)}\,du$$

$$= \frac{3}{2}\int \frac{1}{u^2+1}\cdot 2u\,du - 8\int \frac{1}{u^2+1}\,du + 6\int\left(\frac{1}{u} - \frac{u}{u^2+1}\right)du$$

$$= \frac{3}{2}\int \frac{1}{u^2+1}\,2u\,du - 8\int \frac{1}{u^2+1}\,du + 6\int \frac{1}{u}\,du - \frac{6}{2}\int \frac{2u}{u^2+1}\,du$$

$$= \frac{3}{2}\log(u^2+1) - 8\arctan(u) + 6\log(u) - 3\log(u^2+1) + c$$

$$= \frac{-3}{2}\log((x+1)^2+1) - 8\arctan(x+1) + 6\log(x+1) + c,$$

where c is a constant.