Combinatorics. An Introduction

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Abstract

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0 Preface

This book would not have been written without the help and encouragement of many people.

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I owe a special debt to Don Taylor. Don is largely responsble for the introduction of modern combinatorial ideas to third year students at the University of Sydney. The courses he has developed have greatly influenced the choice and the presentation of many topics in this book.

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1 Introduction

1.1 What is combinatorics?

Broadly, it is about combinations of objects, simple objects: like the natural numbers $1, 2, 3, 4, \ldots$; or subsets of a set; or points and edges.

Because we are concerned with combining separate objects, conbinatorics is often called *discrete mathematics.*

Combining simple objects is very basic: to mathematics, so most mathematics has gone through a combinatorial stage. Even π has combinatorial nature, as we see in the beautiful product of Wallis:

$$\frac{2\cdot 2}{3},\ \frac{1\cdot 2\cdot 4\cdot 4}{3\cdot 3\cdot 5},\ \frac{1\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6}{3\cdot 3\cdot 5\cdot 5\cdot 7},\ \frac{1\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 8\cdot 8}{3\cdot 3\cdot 5\cdot 5\cdot 7\cdot 7\cdot 9},\ \ldots \quad \longrightarrow \quad \pi$$

1.2 What is combinatorics not?

I think that can be summed up in a famous quote of Bertrand Russell, "Mathematics, rightly viewed, possesses . . . supreme beauty, cold and austere, like that of sculpture, without appeal to any part of our weaker nature." Combinatorics is not like that. Combinatorics is more like a mountain meadow, filled with all sorts of interesting and beautiful flowers, which appeals to every part of our nature.

But there is order within the profusion that is combinatorics, and this book attempts to show some of the underlying connections and patterns, not only within combinatorics, but between the discrete and the continuous, between the finite and the infinitesimal. Four of the nine chapters (5,6,8 and 9) are concerenced with pioneering work of Gian-Carlo Rota, whose seriies of papers "*On the Foundations of Combinatorial Theory*", have done the most to bring order out of beautiful chaos. The first of these papers resulted in the award of the 1988 Steele prize to Rota, *for a paper of lasting and fundamental importance.*

This book is based on combinatorics courses given to third year students, at both ordinary and honours level, at the Universrity of Sydney over the last ten years. The theme of the book is the theory of counting. Chapter 1 is concerned with elementary results, including the basic facts about binomial coefficients and Stirling numbers. In Chapter 2 we give a systematic treatment of some of the main techniques used in counting. Chapter 3 is devoted to Pólya theory, which uses group theory to count collections of objects possessing some symmetry.

The combinatorial idenitities which arise when counting a collection in different ways lead naturally to polynomial identitties and in Chapters 4 and 5 and again in Chapters 8 and 9, we explore the interplay between counting and the calculus of polynomials.

In Chpater 6 we give an introduction to Rota's theory of Möbius inversion in partially ordered sets, which brings together the diverse theories of inclusion-exclusion, and Möbius inversion in number theory.

Finally, in Chapter 7, we introduce the notion of Species of Structure due to André Joyal which has great promise and already many achievements, in providing a conceptual interpretation to the theory of generating functions.

This book draws on the mathematics background of third year students. For example, we assume a little matrix theory, some Fourier series in some of the problems of Chapter 4, elementary group theory in Chapter 3 and, in Chapter 7, some experience with functions between finite sets.

"*On the foundations of combinatorial theory I. Theory of Möbius functions.*" Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete, 2 (1964) 340-368.

Based on the classic paper by Joyal, "*Une théorie combinatoire des séries formelles*", Advances in Math. 42 (1981), 1-82.

The choice of topics reflects my interest in the subject and I have made some attempt, not enough, at telling a coherent story, at the expense of the omission of many interesting topics.

1.3 References

Here are some books for further reading.

M. Aigner, *Combinatorial Theory*, Springer-Verlag (1979).

C. Berge, *Principles of Combinatorics*, Academic Press (1971).

D.I.A. Cohen, *Basic Techniques of Combinatorial Theory*, Wiley (1978).

L. Comtet, *Advanced Combinatorics*, D. Reidel (1974).

W. Feller, *An Introduction to Probability Theory and its Applications*, Volume 1 2nd ed. John-Wiley (1957).

H.W. Gould, *Combinatorial Identities – A standardized table of 500 binomial coefficient summations*, Gould (1972).

R.L. Graham, D.E. Knuth, O. Patashnik, *Concrete Mathematics - A Foundation for Computer Science*, Addison-Wesley (1989).

D.E. Knuth, *The Art of Computer Programming, Volume I: Fundamental Algorithms*, 2nd ed. Addison-Wesley (1968).

C.L. Liu, *Introduction to Combinatorial Mathematics*, McGraw-Hill (1968).

J. Riordan, *An Introduction to Combinatorial Analysis*, Wiley (1958).

S. Roman, *The Umbral Calculus*, Academic Press (1978).

G.-C. Rota, *Finite Operator Calculus*, Academic Press (1975).

R.P. Stanley, *Enumerative Combiantorics Volume I*, Wadsworth & Brooks/Cole Mathematics Series (1986).

I. Tomescu, *Introduction to Combinatorics*, Coliet's (1973).

I. Tomescu, *Problems in Combinatorics and Graph Theory*, Wiley-Interscience (1985).

W.A. Whitworth, *Choice and Chance*, Bell (1901).