

### 17.5.5 Constructible numbers

201. Explain why it is not possible to construct (with straight-edge and compass) a line segment whose length is  $2^{1/3}$ .
202. What is  $\sin(30^\circ)$ ?
203. The triple angle formula for  $\sin \theta$  is  $\sin(3\theta) = 3 \sin(\theta) - 4 \sin(\theta)^3$ . Use this formula to decide (with proof) whether it is possible to trisect a  $30^\circ$  angle using compass and straightedge.
204. Suppose that  $a \in \mathbb{R}$  is a constructible number. What can be said about the degree of  $a$  over  $\mathbb{Q}$ .
205. Let  $r \in \mathbb{R}$  be a root of the polynomial  $x^3 + 3x + 1$ . Explain why it is not possible to construct, with straight-edge and compass, a circle of radius  $r$ .
206. Suppose that  $a \in \mathbb{R}$  is a constructible number. What can be said about the dimension of  $\mathbb{Q}(a)$  as a vector space over  $\mathbb{Q}$ ?
207. Is it possible to construct a regular 9-gon with straight-edge and compass? Explain. What about a regular 6-gon?
208. Let  $a \in \mathbb{R}$  be a constructible number. What can be said about the degree of the extension  $\mathbb{Q}(a)$  of  $\mathbb{Q}$ ?
209. Explain why it is not possible to construct with straight-edge and compass a line segment whose length is that of an edge of a cube of volume 5.
210. If a number  $a \in \mathbb{R}$  is constructible what can be said about the degree of the extension  $\mathbb{Q}(a)$  of  $\mathbb{Q}$ ?
211. Is  $\pi$  constructible?
212. Explain why it is not possible to construct with ruler and compass a line segment whose length is the length of an edge of a cube of volume 2.
213. Given that  $\pi$  is a transcendental number, explain why it is not possible to construct (by ruler and compass) a square whose area is the same as the unit circle.
214. Explain why it is not possible to construct (by ruler and compass) a line segment whose length is the length of a side of a cube of volume 2.
215. If a number  $\alpha \in \mathbb{R}$  is constructible what can be said about the degree of the extension  $\mathbb{Q}(\alpha)$  of  $\mathbb{Q}$ ?
216. Explain why it is not possible to construct with ruler and compass a line segment whose length is the length of a side of a cube with volume 2.
217. Is it possible to construct by ruler and compass an angle of  $\pi/9$ ?
218. Is it possible to trisect an angle of  $3t$  degrees if  $\cos(3t) = \frac{1}{3}$ ?