Topic 3. Example 8. Determine the vector, parametric and Cartesian equations of the line passing through the points $P = (-1, 2, 3)$ and $Q = (4, -2, 5)$.

Since

$$\text{the direction of the line is} \qquad Q - P = |4, -2, 5\rangle - |-1, 2, 3\rangle = |5, -4, 2\rangle$$

and

$$P = |-1, 2, 3\rangle \qquad \text{is a point on the line}$$

then the line is the set of points in $\mathbb{R}^3$ given by

$$\{\, |-1, 2, 3\rangle + \, t \cdot |5, -4, 2\rangle \mid t \in \mathbb{R}\}.$$

Parametric equations for the line are

$$\begin{aligned} x &= -1 + 5t, \\ y &= 2 - 4t, \\ z &= 3 + 2t, \end{aligned} \qquad \text{with } t \in \mathbb{R}.$$

Solving for t, the Cartesian equation of the line is

$$\frac{x+1}{5} = \frac{y-2}{-4} = \frac{z-3}{2}.$$

Topic 3. Example 9. Find a vector equation of the "friendly" line through the point $(2, 0, 1)$ that is parallel to the "enemy" line

$$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-6}{2}.$$

Does the point $(0, 4, -3)$ line on the "friendly" line?

Letting

$$t = \frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-6}{2}$$

gives

$$\begin{aligned} x &= 1 + t, \\ y &= -2 - 2t, \\ z &= 6 + 2t \end{aligned} \qquad \text{with } t \in \mathbb{R}, \text{ and} \qquad \{(1, -2, 6) + t((1, -2, 2) \mid t \in \mathbb{R}\}$$

is the set of points in $\mathbb{R}^3$ that lie on the "enemy" line.

The "friendly" line we want is parallel to the "enemy" line and consists of the points

$$\{\, |2, 0, 1\rangle + t\, |1, -2, 2\rangle \mid t \in \mathbb{R}\}.$$

Since $|2, 0, 1\rangle + (-2) \cdot |1, -2, 2\rangle = |0, 4, -3\rangle$ then $|0, 4, -3\rangle$ is on the "friendly' line.