Topic 3. Example 6. Find the area of the triangle in $\mathbb{R}^3$ with vertices $|2,-5,4\rangle$, $|3,-4,5\rangle$ and $|3,-6,2\rangle$. Letting

$$u = |3,-4,5\rangle - |2,-5,4\rangle = |1,1,1\rangle \qquad \text{and} \qquad v = |3,-6,2\rangle - |2,-5,4\rangle = |1,-1,2\rangle$$

and, using that $u \times v = |-1,3,-2\rangle$ (from Topic 3 Example 5), gives

$$(\text{Area of triangle}) = \tfrac{1}{2}\|u \times v\| = \tfrac{1}{2}\|\,|-1,3,-2\rangle\,\| = \tfrac{1}{2}\sqrt{(-1)^2 + 3^2 + (-2)^2} = \frac{\sqrt{14}}{2}.$$

Topic 3. Example 7. Find the volume of the parallelipiped with adjacent edges $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{PS}$, where $P = |2,0,-1|$, $Q = |4,1,0\rangle$, $R = |3,-1,1\rangle$ and $S = |2,-2,2\rangle$. Since

$$\overrightarrow{PQ} = P - Q = |2,1,1\rangle, \qquad \overrightarrow{PR} = P - R = |1,-1,2\rangle, \qquad \overrightarrow{PS} = P - S = |0,-2,3\rangle,$$

then

$$\begin{aligned}(\text{Volume of parallelipiped}) = |\langle P - Q, (P-R) \times (P-S)\rangle| &= \left|\det\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & -2 & 3 \end{pmatrix}\right| \\ &= \left|2 \cdot \det\begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix} - \det\begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}\right| \\ &= |2(-3+4) - (3+2)| = |-3| = 3.\end{aligned}$$