1.12 Composition of functions

Let $f: S \to T$ and $g: T \to U$ be functions. The *composition* of f and g is the function

$$g \circ f \quad \text{given by} \quad \begin{array}{rccc} g\circ f: & S & \to & U \\ & s & \mapsto & g(f(s)) \end{array}$$

Let S be a set. The *identity map on S* is the function given by

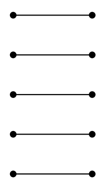
$$\begin{array}{rccc} \mathrm{id}_S: & S & \to & S \\ & s & \mapsto & s \end{array}$$

Let $f: S \to T$ be a function. The *inverse function to f* is a function

$$f^{-1}: T \to S \qquad \text{such that} \qquad f \circ f^{-1} = \mathrm{id}_T \quad \text{and} \quad f^{-1} \circ f = \mathrm{id}_S.$$

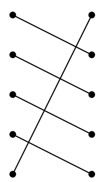
Theorem 1.4. *Let $f: S \to T$ be a function. An inverse function to f exists if and only if f is bijective.*

Representing functions as graphs, the identity function id_S looks like

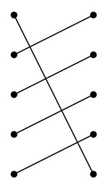


(a) the identity function id_S

In the pictures below, if the left graph is a pictorial representation of a function $f: S \to T$ then the inverse function to f, $f^{-1}: T \to S$, is represented by the graph on the right; the graph for f^{-1} is the mirror-image of the graph for f.

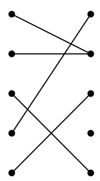


(b) the function f

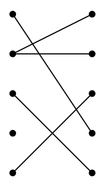


(c) the function f^{-1}

Graph (d) below, represents a function $g: S \to T$ which is not bijective. The inverse function to g does not exist in this case: the graph (e) of a possible candidate, is not the graph of a function.



(d) the function g



(e) not a function