1.13 Cardinality

Let S and T be sets. The sets *S and T are isomorphic*, or *have the same cardinality*

$$\text{if there is a bijective function} \quad \varphi\colon S \to T.$$

Write $\mathrm{Card}(S) = \mathrm{Card}(T)$ if S and T have the same cardinality.

Notation: Let S be a set. Write

$$\mathrm{Card}(S) = \begin{cases} 0, & \text{if } S = \emptyset, \\ n, & \text{if } \mathrm{Card}(S) = \mathrm{Card}(\{1, 2, \ldots, n\}), \\ \infty, & \text{otherwise.} \end{cases}$$

Note that even in the cases where $\mathrm{Card}(S) = \infty$ and $\mathrm{Card}(T) = \infty$ it may be that $\mathrm{Card}(S) \neq \mathrm{Card}(T)$.

Let S be a set.

- The set S is *finite* if there exists $n \in \mathbb{Z}_{\geq 0}$ such that $\mathrm{Card}(S) = \mathrm{Card}(\{1, \ldots, n\})$.
- The set S is *infinite* if $\mathrm{Card}(S)$ is not finite.
- The set S is *countable* if $\mathrm{Card}(S) = \mathrm{Card}(\mathbb{Z}_{>0})$.
- The set S is *countably infinite* if S is countable and infinite.
- The set S is *uncountable* if S is not countable.