Metric and Hilbert spaces, Lecture 9, 13.08.2015

<u>Compactness</u> (topological version)

Let $(X, \mathcal{T})$ be a topological space. Let $E \subseteq X$.

The set E is <u>compact</u> if E satisfies:

every open cover of E has a finite subcover.

In math. E is <u>compact</u> if E satisfies:

if $\mathcal{S} \subseteq \mathcal{T}$ and $E \subseteq \left(\bigcup_{U \in \mathcal{S}} U\right)$

then there exists $\ell \in \mathbb{Z}_{>0}$ and $U_1, U_2, \ldots, U_\ell \in \mathcal{S}$

such that $E \subseteq (U_1 \cup U_2 \cup \cdots \cup U_\ell)$

<u>Important Theorem</u> Let $X = \mathbb{R}$ with the standard topology. Let $E \subseteq X$. Then

E is compact if and only if E is closed and bounded.