Tutorial: Metric and Hilbert spaces, 13 October 2015

(1) Let $T: V \to V$ be a self adjoint linear operator and let v be an eigenvector of eigenvalue λ. Show that $\lambda \in \mathbb{R}$.

(2) Let $T: V \to V$ be a self adjoint linear operator. Let λ and γ be eigenvalues of T with $\lambda \neq \gamma$. Let

$$X_\lambda = \{v \in V \mid Tv = \lambda v\} \text{ and } X_\gamma = \{v \in V \mid Tv = \gamma v\}.$$

Show that X_λ is orthogonal to X_γ.

(3) Let H be a Hilbert space and let

$T: H \to H$ be a compact linear operator.

Let λ be a nonzero eigenvalue of T and let

$$X_\lambda = \{v \in H \mid Tv = \lambda v\}$$

Show that X_λ is finite dimensional.

(4) Let $T: V \to V$ be a bounded linear operator.

(a) Show that T^*T is self adjoint.

(b) Show that if γ is an eigenvalue of T^*T then $\gamma \in \mathbb{R}_{\geq 0}$.