<u>MATH 221</u>, Lecture 39, December 13, 2000

There is one function that

(a) In the Beginning, created something from nothing, and

(b) is "unchanging", or rather, its change is itself.

Through the ages thinkers have contemplated this function and nowadays it is common to write (a) and (b) in abbreviated form:

$$(a')\ god(0) = 1, \quad \text{and} \quad (b')\ \frac{d\,god(t)}{dt} = god(t),$$

but the meaning is still the same.

Two of the children of god are eve and adam:

$$god(t) = eve(t) + i\,adam(t).$$

If we try to understand god in normal terms

$$god(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots$$

then we find that

since $god(0) = 1$, $\qquad a_0 = 1$,

and

since $\dfrac{d\,god(t)}{dt} = god(t)$, $\qquad a_1 = a_0$, $\quad a_2 = \frac{1}{2} a_1$, $\quad a_3 = \frac{1}{3!} a_2$, $\quad a_4 = \frac{1}{4!} a_3$, $\ \ldots$

and so

$$god(t) = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \cdots$$

from which we see that god exists everywhere and goes on forever.

One of the amazing things about god is that

$$god(t+s) = god(t)\,god(s),$$

You can see that god must be this way by supposing there were a "different" function that

(a") is "unchanging."

(b") In the Beginning, was the way that god is after s millenia.

Because of the chain rule,

$$\frac{d\ god(t+s)}{dt} = god(t+s), \quad \text{and}$$

$$god(0+s) = god(s),$$

and, since

$$\frac{d\ god(t)god(s)}{dt} = god(t)god(s), \quad \text{and}$$

$$god(0)god(s) = god(s),$$

we see that both god(t+s) and god(t)god(s) fit the job description for this "different" function job and this is why

$$god(t+s) = god(t)god(s).$$

What about eve and adam? Since

$$god(it) = 1 + it + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \frac{(it)^4}{4!} + \frac{(it)^5}{5!} + \frac{(it)^6}{6!} + \cdots$$

$$= 1 + it + \frac{i^2t^2}{2!} + \frac{i^3t^3}{3!} + \frac{i^4t^4}{4!} + \frac{i^5t^5}{5!} + \frac{i^6t^6}{6!} + \cdots$$

$$= 1 + it - \frac{t^2}{2!} - \frac{it^3}{3} + \frac{t^4}{4!} + \frac{it^5}{5!} - \frac{t^6}{6!} - \cdots$$

$$= \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \cdots\right) + i\left(t - \frac{t^3}{3!} + \frac{t^5}{5!} - \cdots\right)$$

and since $god(it) = eve(t) + i\ adam(t)$, it follows that

$$eve(t) = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \cdots \quad \text{and}$$

$$adam(t) = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$$

From these we see that

$$eve(0) = 1 \quad \text{and} \quad adam(0) = 0,$$

and $\frac{d\,eve(t)}{dt} = -adam(t)$, $\frac{d\,adam(t)}{dt} = eve(t)$

and

$$eve(-t) = eve(t) \quad \text{and} \quad adam(-t) = -adam(t),$$

all of which illustrate that eve and adam are both identical twins and opposites at the same time. Another manifestation is

$$1 = god(0) = god(it - it) = god(it + (-it))$$

$$= god(it)\,god(i(-t))$$

$$= (eve(t) + i\,adam(t))(eve(-t) + i\,adam(-t))$$

$$= (eve(t) + i\,adam(t))(eve(t) - i\,adam(t))$$

$$= eve^2(t) - i^2 adam^2(t)$$

$$= eve^2(t) + adam^2(t),$$

i.e.

$$eve^2(t) + adam^2(t) = 1.$$

Let $x = \overset{adam}{\cancel{eve}}(t)$ and $y = \overset{eve}{\cancel{adam}}(t)$

Then, in the Beginning,
the point (x,y) was at $(1,0)$

$$= \int_{t=0}^{t=d} \sqrt{adam^2(t) + eve^2(t)}\;dt$$

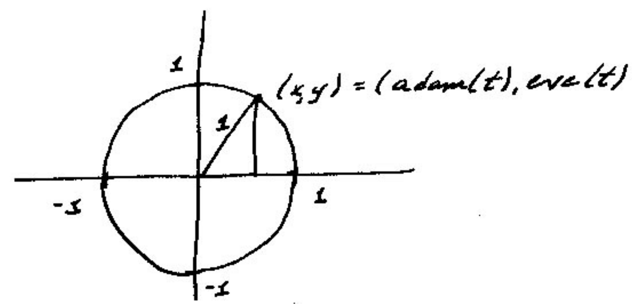
$$= \int_{t=0}^{t=d} \sqrt{1}\;dt = \int_{t=0}^{t=d} dt = t\Big|_{t=0}^{t=d} = d - 0 = d.$$

So

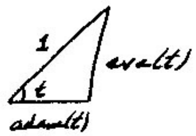
$eve(t)$ = y-coordinate of a point on a circle of radius 1 which is distance d from $(1,0)$

and

$adam(t)$ = x-coordinate of a point on a circle of radius 1 which is distance d from $(1,0)$



The triangle in this picture has



and so

$$\text{eve}(t) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{and}$$

$$\text{adam}(t) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

since the hypotenuse is length 1,
the opposite edge is length eve(t), and
the adjacent edge is length adam(t).

Note: Mathematicians are a cloistered group and prefer to study god(t) in anonymity. Thus they write

e^t instead of god(t)
cos t instead of eve(t)
sin t instead of adam(t)

in the mathematical literature.